Problem of the Month. Solutions: November 2012.

MATHEMATICS

a) Show that there are 2 people in Sigma Camp that have the same number of friends in Sigma Camp.
b) An Island with area 1 km ^{2} is covered by trees – one tree per 1 m ^{2}. Each tree has at most 70000 leaves. Prove that there at least 15 trees with the same number of leaves.

Solution:
a) Let's prove it for any number of people N. Each person can have 0, 1, 2, 3, …, or (N-1) friends, N possible variations. But there are also N students which means that all of the variations are in use and there is a person who has 0 friends and a person who has (N-1) friends which is impossible because then the person who has (N-1) friends is a friend of a person who's got 0 friends (?!).
b) There are 1,000,000 trees on this Island (1 km ^{2} = 1,000,000 m ^{2}).
The tree can have 0, 1, 2, 3,....... 70000 leaves.
Suppose we can't find 15 or more trees that have the same number of leaves. This would the number of trees that have the same number of leaves is less than 15. But if there are less than 15 (i.e. 14 or less) trees have 1 leaf, 14 or less trees that have 2 leaves, 3 leaves, ….., 70000 leaves. But the total number of trees would be less than 14*70001=980,014 which is less than 1,000,000 and contradicts the fact that there are 1,000,000 on the Island.

PHYSICS

A cockroach is dropped on a straight horizontal tree branch which is one meter long. The roach can run to the right or left with a constant speed of 1 meter per minute. When the roach reaches an end of the branch, it falls off. It is clear that no matter what point of the branch the roach lands at initially, the time it takes it to reach the end of the branch is at most 1 minute: 1 minute corresponds to the case when the roach lands at one end of the branch and starts running towards the other end.
Question 1. Imagine that you drop two roaches at the same time. Each of those roaches can travel to the right or to the left, depending on the initial orientation, and the speed is 1 meter per minute. When the roaches meet, they bounce off each other elastically and keeping the same speed run in the opposite directions. When the roach reaches the end of the branch, it falls off. Assuming that two roaches are dropped simultaneously on the tree branch, what is the maximum time one needs to wait to guarantee that both roaches are off the branch?
Question 2. If you start with 2012 roaches what is the longest time you need to wait to guarantee that the branch has no more roaches?

Solution:
When two cockroaches, Bob and Alice bounce off each other, Bob continues on the path of Alice, and vice versa. If we cannot distinguish Bob from Alice, or, we are simply not interested in which roach marches on which path, the situation appears as if they had passed through each other and continued on their initial paths. Hence, the situation looks like all roaches follow their paths, passing through each other: the number of roaches on each path does not change. The longest time for all

roaches to fall off the stick does not depend on their number, and is determined by the longest path for a single one: 1 minute.
Remark: interestingly enough the idea of indistinguishable particles used in the above solution plays a very important role in physics. The concept of indistinguishability of quantum particles is related to physical effects of super fluidity, superconductivity and to the fundamental properties of metals.

CHEMISTRY

To understand the phenomenon of combustion Alex conducted the following experiment: he measured the weight of a sealed glass flask containing some quantity of iron powder, which he then proceeded to heat, and then cool to the initial temperature. He then opened the flask and measured its weight again. Based on the results of his experiment, Alex made a conclusion, that during heating, fire elements enter through the walls of the flask and that these elements can become stable and acquire substance-like properties, such as weight. Later, Mark repeated Alex's experiment with some changes and concluded that Alex's hypothesis (opinion) was false, since…

1. What mistake did Alex commit?
2. How did Mark change Alex's experiment?
3. What conclusions did Mark come to as a result of his own experiments?
4. Calculate the mass of fire-substance, which, according to Alex, entered the flask during heating, given that the flask had a volume of 5L (sealed) and contained 4g of iron powder.

Solution:
1. Alex's mistake was that he opened the flask before weighting. He didn't consider a possibility that the air in the flask can interact with iron. Indeed, when air (or, its component, oxygen) interacts with iron (to form rust, or iron oxide Fe_2O_3), air pressure in the flask drops. When Alex opened the flask, some additional amount of air entered into it, so Alex found that the weight of the *opened* flask *increased.*
2. Mark changed Alex's experiment only slightly. He *didn't open* the flask before weighting. As a result, Mark observed the weight of the flask didn't change after heating.
3. Mark concluded that the increase of mass of iron powder was not due to its interaction with some obscure "fire elements", but due to the reaction between iron and air.
4. The chemical equation is: $4Fe + 3O2 = 2Fe_2O_3$
Atomic mass of iron is 56, atomic mass of oxygen is 16. That means that 56*2=112 g of iron react with 16*3=48 g of oxygen. If 112g of iron react with 48 g of oxygen, then 4 g of iron react with 48*(4/112) =1.71g of oxygen, which is equal to the maximal increase of the weight of 4g of iron after heating. However, we do not know if the amount of oxygen in the 5L flask is sufficient to react with 4g of iron. To check that, we need to calculate the amount of oxygen in 5L of air. We know that 1 mol (32g) of oxygen occupies a volume of approximately 22.4L at normal temperature and atmospheric pressure. Since oxygen constitutes only 20% of air mass, 22.4L of air contain 32*0.2=6.4 g of oxygen. Accordingly, 5L of air contain 6.4*(5/22.4) =1.49 g of oxygen, whereas 1.71 g of oxygen is needed to convert all 4 g of iron into iron oxide. That means all oxygen in the flask will react with iron after heating (and even some amount of non-reacted iron will remain). Accordingly, the increase of the mass of iron (the mass of Alex's mystical "fire-substance") will be equal to the amount of reacted oxygen, 1.49g.
Comment. Actually, in this problem we described a real story of two chemists, a British chemist Robert Boyle ("Alex") and a Russian chemist Mikhailo Lomonosov ("Mark").

BIOLOGY

It is known that some animals live in families. Some live in big groups. Others live mostly by themselves. Animals have to find the right way of living together so they can stay safe, take care of their babies, and find food.

Groups of animals (these are called "aggregates") may be either temporary (animals group only for specific purposes) or permanent. For example, many wild and domestic animals live in herds.

Please, provide other examples of permanent groupings of different animal species and explain why these groupings might be important for survival.

Give examples of temporary groups. Why do you think such temporary groupings exist?

Feel free to illustrate your answer with images.

Solution:

The question of why animals group together is one of the most fundamental questions in sociobiology and behavioral ecology.

The primary functions of aggregation appear to be feeding and defense. A general theory explaining why individuals should prefer to aggregate was first proposed by the Briton W.D. Hamilton, one of the most famous evolutionary biologists of the 20th century.

Reasons for animals to live in groups:

1. ***"Safety in numbers":*** In a large herd, a predator can only take a few individuals. By staying with others of their own kind, each animal is lowering its personal odds of being chosen. Also, there are more eyes watching for predators. This is part of the reason why birds form mixed foraging flocks.
2. ***"It takes a village"***: Many animals group together to raise their young. For some, this is a subset of #1 above - if all the young are being born at the same time and place, each individual's risk of being a prey is lower. For others, it is a way to pool resources, with adults sharing the protection, defense or feeding of the youngsters. In many bird species, young birds live with their parents in family groups for a season or two before trying to mate on their own - they use this time to learn parenting skills, and their contributions to nestling care increases their parents' success as well.
3. ***For predators:*** Greater success in finding, killing or defending prey items. Wolves are actually more successful as lone hunters than in a pack, but cannot defend their kills from bears and even raven when they are alone.
4. ***Mate finding:*** Very large territory makes finding mates difficult. Some animals live together in small groups because finding one another across large territories with low population densities is hard.

Read more:

http://www.tutorvista.com/content/biology/biology-iv/biotic-community/group-formation.php#
http://en.wikipedia.org/wiki/Collective_animal_behavior
http://www.nature.com/scitable/knowledge/library/cooperation-conflict-and-the-evolution-of-complex-13236526

Group classification based on social behavior:

1. ***Gregarious groups*.** Animals don't interact with one another socially, but they stay together because it is advantageous. One reason is that there are more eyes and ears to watch for predators. There is also some safety in numbers against lone hunters. Often the individuals congregate at the same place because of similar interests. E.g. a group of brown pelicans may fish at the same place because there is a lot of fish there.

2. ***Social groups***. E.g. a troop of baboons, with a social hierarchy. The most dominant alpha individual has an unquestionable priority over other troop members. Besides the protection of a group, a social group reduces fights between individuals because of an established hierarchy. Each animal knows its rank within the group, so there is less fighting. In these cases disputes are usually settled by posturing.

3. ***Eusocial groups*,** such as naked mole rats, bees and ants. There is a division of labor and only a few individuals within a colony may breed. These groups are super organisms. Each individual within a group is equivalent to a body cell of an individual organism. These groups usually cannot survive as single individuals.

Examples of permanent aggregates:

Insects: swarms, colonies
Fish: shoals, schools
Birds: flocks, colonies
Mammals: prides (lions), herds (cows, elephants, buffaloes, deer), troops (apes and monkeys), packs (wolves), harems (baboons, zebras).

Read more: http://www.bath.ac.uk/bio-sci/biodiversity-lab/teaching/evolution/L3%20-%20Living%20in%20groups.pdf

Temporary aggregation:

The simplest kind of animal society is the temporary aggregation. Such groups occur widely in the animal kingdom and were extensively studied by W. C. Allee.

In this case animals become gregarious temporarily, either in the time of breeding, migration or hibernation.
Locusts and many solitary birds form large groups during migration.
Frogs become gregarious during breeding season
Snakes hibernate in groups for warmth;
Birds and mammals form groups for mating and sexual reproduction and/or for parental care.

The Costs and Benefits of Group Living:

Group-living typically provides benefits to individual group members. But living in groups may also confer costs to members. As individuals aggregate, they become more conspicuous to predators and competition for food can increase. Therefore, when deciding to join a group, individuals must weigh the cost-benefit ratio of living solitarily versus with others. Other benefits of group-living may include receiving assistance to deal with pathogens (i.e., grooming), easier mating opportunities, better conservation of heat, and reduced energetic costs of movements. Other costs of group-living may include increased attack rates by predators, increased parasite burdens, misdirected parental care, and greater reproductive competition.