PROBLEM OF THE MONTH

POM

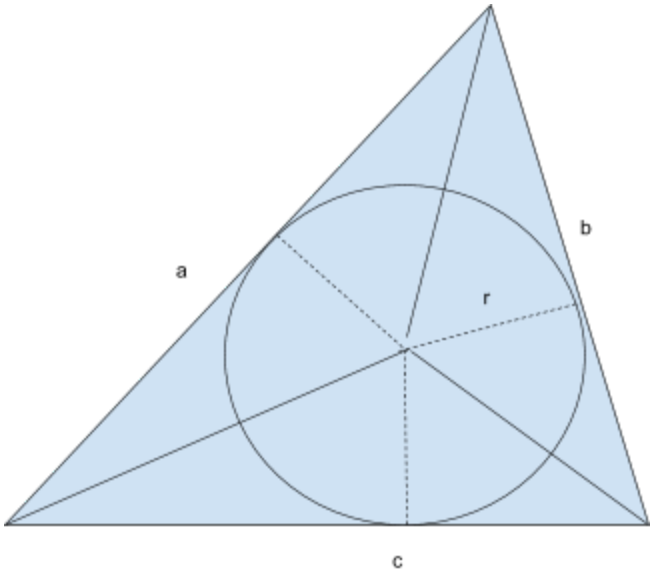
2017-2018

November, 2017

MATHEMATICS

5 points: A circle is inscribed in a triangle. The perimeters of the circle and the triangle are 1cm and 10 cm, respectively. Find the area of the triangle.

Hint: Look at the picture.



10 points: A sphere is inscribed in a polyhedron so that it touches all of its faces. The volume of the sphere is 9 cm^3. The surface area of the polyhedron is 50 cm^2. What is its volume?

Hint: same as for 5 pt problem. You'll need to imagine a similar construction in 3D. Note that volume of a pyramid is Bh/3 (B is the area of its base which may have any shape), h is its height.

PHYSICS

This month Physics problems are on the mechanical work and mechanical energy conservation. You might find the following links useful.
https://en.wikipedia.org/wiki/Work_(physics)
http://hyperphysics.phy-astr.gsu.edu/hbase/ke.html
http://hyperphysics.phy-astr.gsu.edu/hbase/pegrav.html
http://hyperphysics.phy-astr.gsu.edu/hbase/frict.html

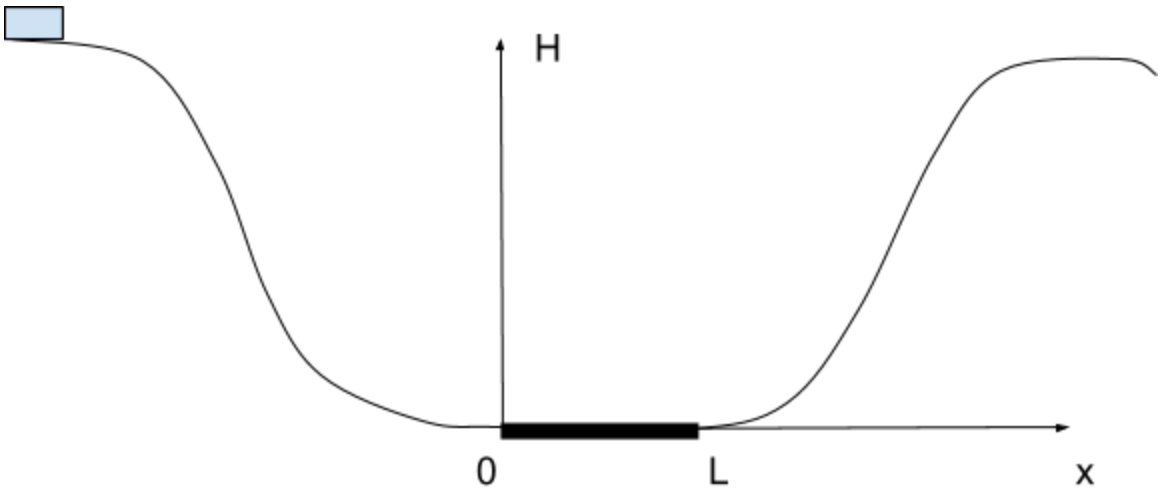
5 pt

A spring gun using a massless spring with the spring constant $k = 100\ N/m$ is loaded with the ball of the mass $m = 0.01\ kg$, so that the maximal compression of the spring is $A = 10\ cm$. What is the maximal height reached by the ball if the gun shoots strictly vertically. Neglect friction and air resistance. Assume that the free fall acceleration $g \approx 10\ m/s^2$.

Hint: Use the conservation of energy.

10 pt

A small block is sliding from a hill of the height $H = 5m$ down the frictionless slope towards the similar hill. There is a small horizontal patch of the surface with kinetic friction coefficient $\mu = 0.3$ of the length $L = 1m$ between the hills. At what position will the block stop?



Hint: Find the work done by friction force.

CHEMISTRY

5 points:

DNA means “deoxyribonucleic *acid*”, but if you take a commercially available solution of DNA and measure its pH, the solution will be nearly neutral. Similarly, cell nuclei are full of densely packed DNA, but the media is neutral there. Why does it happen, and is DNA a real acid?

Hint:

Classical (“Bronsted’s”) acids always contain a constant part (hydrogen atoms) and a variable part (this variable part may be huge and complex, especially in organic acids, and it is this part which makes acids different from each other). In solution, hydrogens usually dissociate from the acid molecule, and most biochemists prefer to (...continue by yourselves).

10 points:

In a ***L'Alpagueur*** movie, Jean-Paul Belmondo’s hero uses a nitrous oxide, N_2O (“laughing gas”) to put mafia bosses to sleep. To do that, he drilled holes in the floor of a trailer where the mafia meeting took place and directed a stream of laughing gas there. The gas cylinders he used for narcotizing mafia are shown on the figure 1 (he had two). The size of the trailer can be estimated based on the picture of its interior (Fig. 2).

Please, tell if the amount of the gas was sufficient to bring mafia bosses into an unconscious state.

To answer this question, assume that the gas cylinders are standard cylinders used for medical purposes, and they are full. Estimate their actual size from the attached picture. The concentration of N_2O in air that is necessary to provide a desirable effect can be googled.

If, according to your estimate, the amount of N_2O is too small or too big, how many cylinders of that size should the hero have used to achieve a desirable result?

Fig. 1. The cylinders.

Fig. 2. The trailer. For your estimate assume one half of the trailer is seen on the picture.

Hint:

English Wikipedia contains all essential information needed to answer this question. Due to its low boiling point value, N_2O is in a gaseous state in cylinders. Calculate the gas amount assuming it obeys ideal gas laws (when the gas volume decreases twice, the pressure doubles).

BIOLOGY

5 points:

There are numerous examples of synchronized behavior in nature: claw waving (fiddler crabs), synchronized respiration (honey bees), or chewing (termites). One of the famous cases is synchronized flashing in fireflies frequently seen in Asia and North America. In the United States it is attributed to the rover firefly (*Photinus carolinus*) and can be observed in early June in, for example, Great Smoky Mountains National Park. Males emit well-synchronized periodic flashes of bright light. It is considered to be a mating mechanism to attract females. Why do these bugs need "dark intervals" between flashes? Interestingly, not all fireflies need dark intervals. For example, European fireflies emit constant light with no flashes. What can be the reason for such a difference?

10 points:

Imagine that there is a rare disease which is caused by a mutation in a single autosomal gene. It is known that the disease develops only in individuals who have two mutant alleles (homozygots). They are unable to have children. Heterozygous (one mutant and one normal alleles) individuals are healthy and can have children. The incidence of this disease is 1 in 100,000. Do you think that this illness will completely disappear from the population in 100 generations? Why or why not?

COMPUTER SCIENCE

- You can write and compile your code here: http://www.tutorialspoint.com/codingground.htm
- Your program should be written in Java or Python
- No GUI should be used in your program: eg., easygui in Python. All problems in POM require only text input and output. GUI usage complicates solution validation, for which we are also using *codingground* site. Solutions with GUI will have points deducted or won't receive any points at all.
- Please make sure that the code compiles and runs on http://www.tutorialspoint.com/codingground.htm before submitting it.
- Any input data specified in the problem should be supplied as user input, not hard-coded into the text of the program.
- Submit the problem in a plain text file, such as .txt, .dat, etc. **No .pdf, .doc, .docx, etc!**

5 points:

You need to write a program that analyzes the results of a Tic-Tac-Toe game.

The program should enter a Tic-Tac-Toe board as a 3x3 array of X's and O's from the input. The program should print whether X won, O won, it was a draw or the position on the board is not possible.

10 points:

You have an NxN matrix, each cell of which contains an integer number (could be zero or negative). A ***path*** in such matrix can start in any cell and then go through adjacent cells to the right or down. A path could be of any length (obviously, the longest path would be 2*N-1 cells long).

3 possible paths are shown in red in the diagram below:

4	3	0	7	4
4	-7	1	15	-2
0	5	8	6	0
-5	11	4	-1	1
1	4	5	3	12

A ***value*** of the path is a sum of all the integer numbers contained in the cells along the path. For example, all the paths in red in the diagram above have value of 10. Your task is to find all the paths in the given matrix with a specific value.

Your program should enter the matrix's dimension and all the cell values from input, as well as enter the target path value. For each path of the given value the program should print the values of the constituent cells.