PROBLEM OF THE MONTH

2014-2015

POM

NOVEMBER, 2014

MATHEMATICS

5 points:

Is it possible to place 4 soccer players on a soccer field so that pairwise distances between them are equal to 1, 2, 3, 4, 5, and 6 meters?

Solution:

Answer: yes

One should place them along a straight line at positions 0, 2, 5, 6. Then their distances from each other are exactly 1, 2, 3, 4, 5, 6. One can also prove that it is impossible to place the players not along a straight line.

10 points:

Given an equilateral triangle ABC find all points M on the plane such that both triangles ABM and ACM are isosceles.

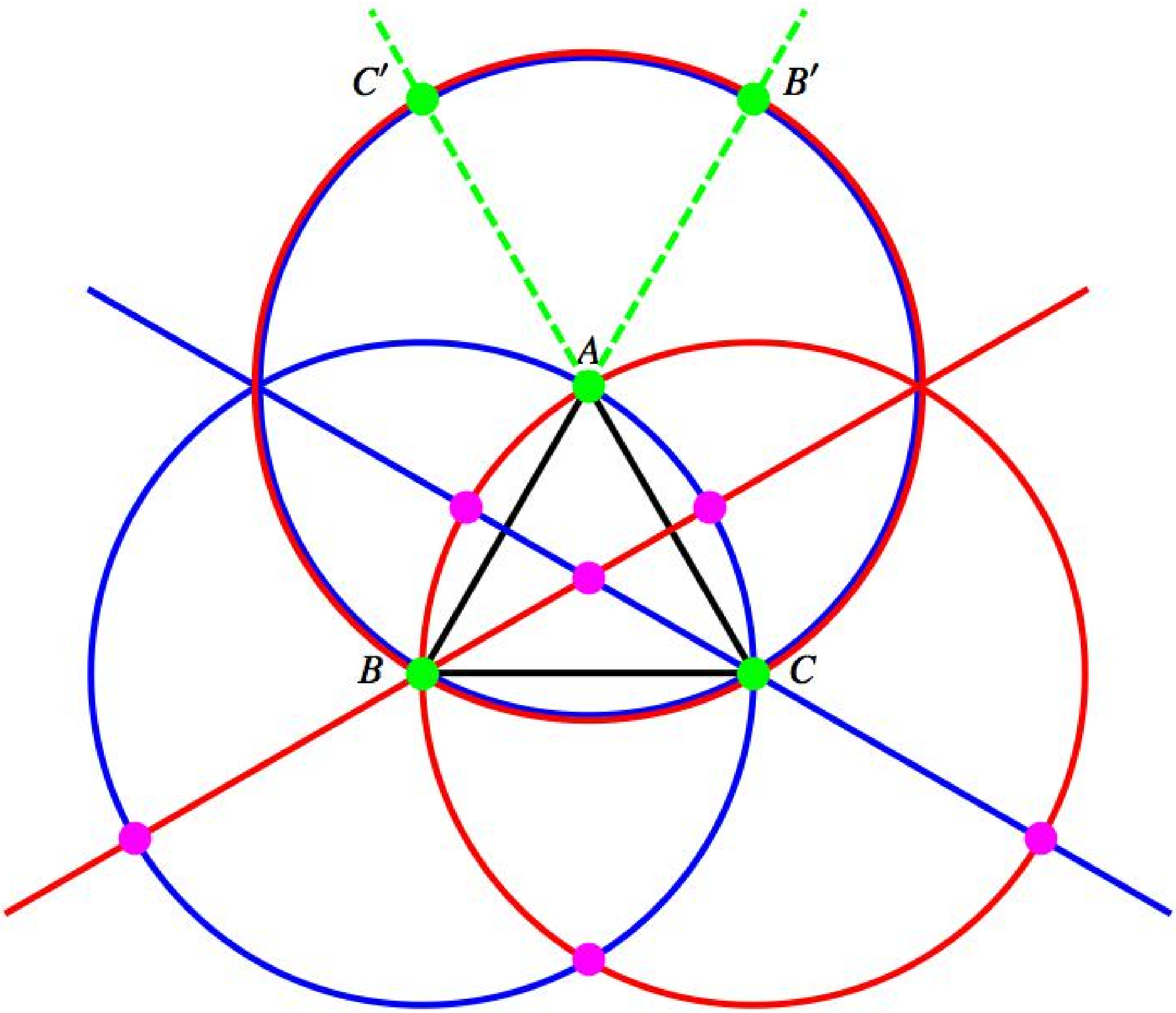
Solution:

Answer: the circle with the center A and radius AB (with five excluded points A, B, C, A', B') and additional six points shown in magenta in the figure.

Let us find all positions of M making the triangle ABM isosceles. If AB is a base of the triangle all positions of M can be found by constructing a perpendicular bisector of AB (blue straight line in the figure). If AB is a side of ABM the point M should belong either to the circle with the center B and radius BA or to the circle with the center A and radius AB. Both circles are shown in blue in the figure. We conclude that the point M should be colored blue in the figure below.

We repeat the same arguments for the triangle ACM and find that the point M should belong either a straight line or any of two circles shown in red in the figure.

Let us now find all points that are colored both red and blue. These are the points belonging to the circle with the center A and radius AB (it has double coloring) and additional six points (labeled by magenta color in the figure). If M is on the straight line AB or AC one of the figures ABM or ACM is not a triangle and therefore, 5 points A, B, C, B', C' shown in green in the figure should be excluded from the solution.



PHYSICS

5 points:

A person stays on scales in the elevator, which is initially at rest. The elevator begins moving upwards, and first moves with a constant acceleration for two seconds. After that, it moves up with a constant speed. Before coming to rest, the elevator decelerates for 3 seconds, again with a constant acceleration.
The reading of the scales was 60 kg during acceleration, and 40 kg during deceleration. What is the mass of the person?

Solution

If a is acceleration during the first two seconds, the acceleration during the 3 seconds of stopping must be $a' = -2a/3$. The reading on scales is proportional to the normal reaction force N acting on a person. During acceleration, by 2nd Newton's Law $ma = N - mg$ (Here m is the mass of the person, and g=9.81m/s^2 is the gravitational acceleration). Therefore

$$mg + ma = N = g \times (60kg)$$

Similarly, during the deceleration

$$mg - \tfrac{2}{3}ma = g \times (40kg)$$

In these two equations, acceleration a is unknown. It can be excluded if we multiply Eq 1 by 2, Eq 2 by 3, and add them together:

$$5mg = 3g \times (40kg) + 2g \times (60kg)$$
$$m = (3 \times 40kg + 2 \times 60kg)/5 = 48\ kg$$

10 points:

The maximum speed with which a car can stay on a flat, horizontal circular road of radius R=50m without skidding, is v_0=20m/s. Find the minimum time it will take this car to stop on a straight road, if it moves with speed v_1= 30m/s.

Solution:

When a car makes a turn, its centripetal acceleration is $a = v_0^2/R$, which is caused by static friction force. The maximum force of static friction on a flat surface is $mg\mu_s$ (here μ_s is coefficient of static friction and mg is the weight of the car).

Now note that the friction force is also responsible for braking. In a real situation. a car often slides during braking, and therefore kinetic friction would seem more relevant than static one. However, the static friction is higher, so the skillful driver or car's electronics can make sure that the car does not slide. If that is the case, the maximum possible deceleration of the car is equal to the centripetal acceleration in the previous example, $a = v_0^2/R$ .The minimal stopping time is therefore

$$t = v_1/a = \frac{v_1 R}{v_0^2} = 3.75s$$

CHEMISTRY

5 points:

Plutonium is a radioactive element, which is extremely harmful to humans not only due to its radioactivity, but also due to its high toxicity. Imagine you took 1 gram of plutonium (III) chloride and dissolved it in 10 L of water. Obviously, this solution is extremely dangerous and toxic. Then you took 100 mL of this solution and added 9.9 L of pure water to it. The solution obtained is less toxic, but it is still toxic. You took 100 mL of this solution and again added 9.9 L of water to it. How many times do you need to repeat this operation until the expected number of Pu atoms in the final solution is no more than 1?

Solution

To solve this problem, we need to know two basic things:

- 1 gram of hydrogen contains $6*10^{23}$ atoms;
- atomic weights of all elements can be found in the Periodic table.

If we look at the Table, you will see plutonium's weight is 244 atomic mass units (or daltons, Da), the weight of chlorine is approximately 35.5 Da, and hydrogen's weight is 1 Da. That means a single molecule of plutonium (III) chloride weighs 244+35.5*3=350 Da, and, therefore, 1 gram of plutonium (III) chloride contains 349 times less particles than there are atoms in 1 gram of hydrogen, or $6*10^{23}/350 = 1.7*10^{21}$ atoms. In other words, if we take 1 gram of $PuCl_3$, dissolve it in 10 L of water, than there will be=$1.7*10^{21}$ atoms of plutonium in 10 L of this solution. When we take 100 mL of this solution (there are $1.7*10^{19}$ atoms of plutonium (100 times less) in 100 mL of it) and add 9.9 L of water to this solution, you dilute it 100 fold. The first dilution gives $1.7*10^{19}$ atoms of plutonium per

10 L, the second dilution gives $1.7*10^{17}$ atoms, and so on. Obviously, to get approximately one atom of Pu, you need to repeat this operation 11 times.

10 points:

Imagine you are in a chemistry laboratory with the standard equipment. The only chemicals available to you are carbon (charcoal) and another element of your choice. You can use any equipment you want, as well as an unlimited amount of water, air and electricity. Catalysts, which are not consumed in your reactions, are also considered as equipment.
Using these materials and equipment, prepare as many different chemical compounds as possible. “Prepare” means to draw the equation, or a series of equations describing a chemically plausible chemical reaction leading to a desired substance.

Solution

There are many solutions for this problem. The simplest way to prepare a large variety of different compounds is to take some metal. You may take, for example, calcium.
Compound 1, carbon dioxide.
$C + O_2 \rightarrow CO_2$ (combustion of carbon)

Compound 2, carbonic acid .
$CO_2 + H_2O \rightarrow H_2CO_3$ (dissolution of carbon dioxide in water, formation of carbonic acid)

Compound 3, calcium oxide.
$Ca + O_2 \rightarrow CaO$ (combustion of calcium)

Compound 4, calcium nitride.
$Ca + N_2 \rightarrow Ca_3N_2$ (calcium nitride, along with calcium oxide, forms when calcium burns in air)

Compound 5, calcium hydroxide.
$CaO + H_2O \rightarrow Ca(OH)_2$ (reaction of calcium oxide with water)

Compound 6, calcium carbonate.
$Ca(OH)_2 + CO_2 \rightarrow CaCO_3 + H_2O$ (bubbling of carbon dioxide through a solution of calcium hydroxide)

Compound 7, calcium bicarbonate.
$CaCO_3 + CO_2 + H_2O \rightarrow CaHCO_3$ (when you continue bubbling of carbon dioxide through a suspension of calcium carbonate it converts into calcium bicarbonate)

Compound 8, carbon monooxide.
$CO_2 + C \rightarrow CO$ (when carbon dioxide passes through hot charcoal, carbon monooxide forms. That is why fireplaces are dangerous, and every home has a carbon monooxide sensor).

Compound 9, hydrogen.
$Ca + H_2O \rightarrow Ca(OH)_2 + H_2$ (reaction of calcium with water; by the way, it is a second method for preparation of calcium hydroxide)

Compound 10, ammonia.
$Ca_3N_2 + H_2O \rightarrow Ca(OH)_2 + NH_3$ (reaction of calcium nitride with moisture; by the way, it is a third method to prepare of calcium hydroxide)

Actually, that is quite enough for 10 points, however, you may propose many more reactions, especially taking into account you are allowed to use any catalyst you want. For example, you can produce calcium carbide:
$CaO + C \rightarrow CaC_2$ (sintering calcium oxide with carbon)

and use it to produce acetylene gas:

$CaC_2 \rightarrow Ca(OH)_2 + C_2H_2$ (sintering calcium oxide with carbon)

By preparing acetylene, you open a door to a realm of organic chemistry, which means you can produce virtually INFINITE number of compounds (remember, you have an unlimited access to any catalyst you want. For example, you may prepare acetaldehyde

$C_2H_2 + H_2O \rightarrow CH_3CHO$ (in the presence of mercury salts as a catalyst acetylene and water combine to produce acetaldehyde),

and then to convert it to ethyl alcohol

$CH_3CHO + H_2 \rightarrow CH_3CH_2OH$ (in the presence of noble metal catalysts)

and so on. Of course, we do not expect you to know all of that, it is just a demonstration of how many compounds can you prepare.

BIOLOGY

5 points:

In 1774 - 1775, British scientist Charles Blagden conducted a series of experiments in a "super-sauna", a heated chamber he built to measure what is the highest temperature a living creature could bear. He himself, his assistants, and his dog appeared to be capable of sustaining a temperature as high as 127 degrees Celsius for 20 minutes. To prove that there was no mistake in temperature measurements (in those time thermometers were not as precise as modern devices are) he took twenty eggs and a fat juicy steak with him into his "super-sauna", and he found that after twenty minutes the eggs were roasted quite hard, and the meat was rather overdone.

However, neither the scientists nor the dog were negatively affected by this extreme heat. From this experiment, Blagden concluded that human and animal organisms possess some mechanisms that allow them to fight extreme heat and to maintain a constant body temperature.

Please answer the following questions:

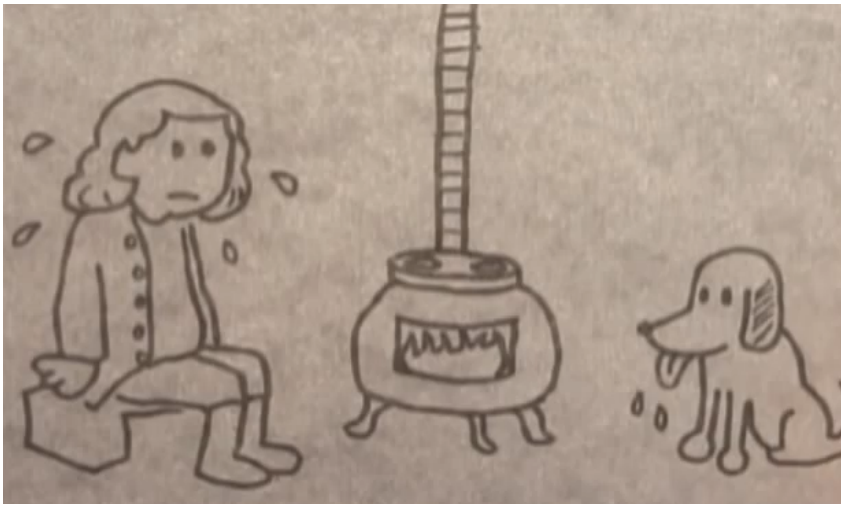
1. What is the mechanism that allows a human body to get rid of excess heat?
2. Do dogs utilize the same mechanism? If not, what mechanism do dogs use for thermoregulation?
3. What other mechanisms of thermoregulation are used by animals?

Solution

Every warm-blooded animal periodically comes across the problem of having too much excess body heat. The prevalence of this issue is determined first and foremost by an animal’s metabolic rate, as well as by specific features of its anatomy. A number of other contributing factors can also play a role. As a result of evolution, animals developed various ways to protect themselves from overheating.

Humans are capable of withstanding high temperature, such as those in the super sauna designed by Charles Blagden, because the human body is adapted to use evaporative cooling to get rid of excess heat. Most of our body is covered by sweat glands - eccrine sweat glands, to be specific, which release water and salt in a process we commonly refer to as sweating. In fact, heat convection is the primary function of sweating in humans, although it can take on other forms and have other functions in mammals as we will see a little bit later. When we sweat, we release a dilute solution of water and sodium chloride onto the surface of our skin. The heat coming from the body raises the temperature of this layer of moisture, which begins to evaporate and draw the excess heat away with it.

Although we don't encourage you to repeat Mr. Blagden's experiments, we do encourage you to go here to watch and learn a little more about his observations: https://www.youtube.com/watch?v=lqwPS6wJN-c. It is really fun!



A different and prominent example of a cooling system that conducts excess heat out of the body is apparent in dogs. Thanks to their thick fur, dogs are better off in the cold, rather than in a hot environment. Because of its insulating qualities, the fur is good for keeping the dog's body temperature stable, but during periods of physical exertion this actually becomes a disadvantage. Dogs do sweat, but they do so through specialized glands known as apocrine sweat glands, which us humans also have in our armpits, nostrils, mammary glands, and a few more locations, and which release mostly fat-based secretions designed to keep the fur healthy and water-resistant. Since fat evaporates poorly, sweating doesn't do much to help dogs in getting rid of body heat.

To get rid of excess heat from their bodies, dogs use *polypnea*, which is also referred to as simply panting. As the dog pants, its excess heat escapes as moisture evaporates from the surface of the dog's tongue and mouth epithelia. The higher the dog's temperature, the more it sticks out its tongue to increase this evaporation. In addition to the tongue and mouth, moisture can escape from the epithelia of the airways, while hot air exhaled from the lungs also carries away heat, further cooling the dog down. If the body temperature is higher than that of the outside air, the dog will inhale through the nose,and exhale through the mouth. As the cooler and drier air passes through the dog's nasal passages, airways and lungs, it will cool down its body by taking moisture and heat away from it, and this newly heated and wet air will then be exhaled through the mouth. This breathing process that is used for cooling down is known as respiratory convective heat exchange.

In addition, dogs can release a small part of their excess heat through the sweat glands on the soft pads on their paws, or other areas that are free of fur. However, this type of heat exchange is not enough to truly protect the furry guys from overheating. Respiratory convection through panting is the only thing dogs can rely on to draw heat away from the entire body.

What other mechanisms of thermoregulation are used by animals?
Evaporation from the surface of the body to increase heat exchange. Many mammals use sweating and moisture evaporation from the airway epithelia. Birds do only the latter, since they lack sweat glands.
Vascular reactions - expansion of blood vessels located close to the surface of the skin, which facilitates release of heat from the body. Birds rely on vascular thermoregulation in areas on their bodies that lack feathers (areas known as *apteria*). The comb on a chicken is a good example of such an area, and is generally cooler than other parts of the chicken's body. Lizards can increase the rate of blood flow through their vasculature when they need to get rid of heat.
Hyperventilation in insects. To cool down, insects evaporate moisture by opening their breathing pores. This is characteristic behavior for cicadas.
Irrigation of the beehive. The temperature inside a beehive can be kept lower than that of the outside by ventilation and evaporation of water that bees purposefully bring into the hive.
Use of shelter. Many smaller animals cannot afford to lose water to evaporative cooling, but they often do not need it, since they take cover from the midday sun either in the shade or in cooler holes and burrows.
Desert dwelling rodents do not sweat, but they do have an "emergency" thermoregulatory mechanism - they **release large amounts of saliva** upon heat stress. The saliva moistens the fur on their lower jaws and on the neck, providing temporary relief if the body temperature approaches dangerous levels. Some reptiles such as turtles also salivate for thermoregulation. In addition, when the turtles' body temperature becomes too high, they urinate on their hind legs. In fact, the presence of a large urinary bladder in desert dwelling turtles puzzled biologists for quite some time, until they discovered that urine was used for thermoregulation and as chemical protection against predators.

10 points:

It is common in zoology and ecology to divide animals into carnivores and herbivores. This classification allows us to distinguish animals that rely on thoroughly different food sources.

1. What are the physiological differences between carnivores and herbivores?
2. How do carnivores and herbivores differ in the enzymes that they make?
3. How do you think omnivores (animals that can eat both meat and plants) are different from the above mentioned groups?

1. What are the physiological differences between carnivores and herbivores?

CARNIVORES (wolves, dogs, cats). Carnivore means 'meat eater' (Latin carne meaning 'flesh' and vorare meaning 'to devour') and classifies animals whose diets consist mainly of meat – such as dogs and cats.

The anatomical features of carnivores are:

SHARP TEETH (designed for slicing meat, not grinding plants). Carnivores have elongated teeth designed for tearing and killing prey. Their molars are triangular with jagged edges that function like serrated-edged blades that give a smooth cutting motion like the blades on a pair of shears. Their facial muscles reduced to allow wide mouth gape. Carnassials on the upper and lower jaw fit together like scissors allowing these teeth to tear apart flesh.
EYES tend to be in towards the front of their heads – although this results in a narrower field of view, it allows them to see with binocular vision, giving them the good depth perception needed to catch their prey.

JAWS MOVE VERTICALLY unlike herbivores and omnivores that grind their food by side to side chewing, the jaws of dogs and cats operate vertically to provide a smooth cutting motion, and open widely to swallow large chunks of meat.

SHORT, SIMPLE & ACIDIC DIGESTIVE TRACTS. Protein and fat from animal source are quickly and easily digested – hence the short digestive system of dogs and cats. The ability of carnivores to secrete hydrochloric acid is also exceptional. To facilitate protein breakdown and kill the bacteria found in decaying meats, they are able to keep their gastric pH around 1-2.

NO AMYLASE IN SALIVA. As amylase is not present in saliva, the burden is entirely on the pancreas to produce the amylase needed to digest carbohydrates.

HERBIVORES (cows, sheep). Herbivores eat plants, not meat. So it’s no surprise that their anatomical features are adapted to process carbohydrates and other nutrients produced by plants. Anatomical features common to herbivores confirm their adaptation for a plant-based diet.

The anatomical features of herbivores are:

Herbivores such as rodents and rabbits tend to have very well-developed SQUARE FLAT MOLARS that provide an ideal grinding surface to crush and grind plants (but not meats). A lower jaw with a pronounced sideways motion facilitates the grinding motion needed to chew plants. Their facial muscles well-developed to allow extensive chewing.
EYES. Because they are often prey for other animals, they tend to have their eyes on the sides of their heads giving them a wider field of view so they can detect their prey early enough to get away.

LONG DIGESTIVE TRACTS up to 10 times their body length are needed due to the relative difficulty with which plant foods are broken down. Herbivores have significantly longer and much more elaborate guts than do carnivores.

CARBOHYDRATE-DIGESTING ENZYMES IN SALIVA. AMYLASE is a digestive enzyme in saliva that helps in digesting carbohydrates. Herbivores methodically chew their food to ensure the thorough mixing with amylase.

2. How do carnivores and herbivores differ in the enzymes that they make?

The process of food digestion is a biological process that consists of a step-by-step breakdown of food polymers with various enzymes, until these polymers are broken down into monomers as they move through the different stages of the digestive tract, which starts at the mouth and goes all the way down to the small and large intestines.

The step by step digestion process relies on breaking food down into more and more basic components. This requires some variety of enzymes. The enzymes found in the digestive tract can be classified as follows, based on where they come from:

1. Enzymes secreted by the digestive organs themselves the mouth, stomach, pancreas, and liver.
2. Enzymes coming from the microorganisms that reside in the different areas of the digestive tract (and we have lots of these)
3. Enzymes that come from plant that we use as food sources.

The diversity of food sources leads to having a very diverse enzyme arsenal to break all of the different and complex food sources down into most basic sugar, protein, and fat components. These enzymes are grouped, almost intuitively, as following (bear with us for the latin origins of these names): amylolytic - which break down carbohydrates of varying complexity into simpler sugars, proteolytic - which break down proteins into smaller pieces (known as peptides), and lipolytic - which break down fats. If we go into more detail, we will find the following specific examples, which mostly come from these three principal groups, at work in the digestive tract:

- Proteins are digested by proteases (endopeptidases and exopeptidases).
- Carbohydrates are digested by carbohydrases (amylase, glucosidase, invertase, galactosidase).
- Fats are digested by carboxylesterases (lipase, phospholipase, cholinesterase).
- Nucleic acids (which present a category of their own) are digested by nucleases (ribonuclease, deoxyribonuclease)

The point of all these enzymes is to break down their specific substrates into the simplest, monomeric units of a nutrient type that can be used for the body for its own metabolism. Let's look at the final products of digestion of our enzymes:

- Proteins are broken down, or hydrolyzed, into amino acids.
- Fats are broken down into fatty acids and glycerin.

- Carbohydrates are broken down into simple hexoses, primarily - glucose.
- Nucleic acids are broken down into purines, pyrimidines, ribose, deoxyribose, and phosphate.

The spectrum of enzymes can change with the age of an animal, and is known to be strongly dependent on the feeding habits and food sources for that animal.

Carnivorous animals feed primarily on **protein-based sources.**	Herbivorous animals feed primarily on plant food sources (i.e. **carbohydrates**).
Carnivores rely primarily on **proteases.**	Herbivores rely primarily on **carbohydrases.**
The enzyme spectrum of carnivores is represented mainly by the following enzymes: **Trypsin, Pepsin.**	The enzyme spectrum of herbivores is represented mainly by the following enzymes:**Amylase, Maltase, Ptyalin (aka Salivary Amylase)**

3. How do you think omnivores (animals that can eat both meat and plants) are different from the above mentioned groups?

Omnivores (from Latin: omne all, everything; vorare to devour) are evolved to eat both plants and animals.As general feeders, omnivores are not specifically adapted to eat meat or plant material exclusively. Euryphagous animals, or euryphages, are animals that have changed and expanded their original food sources over the course of evolution in order to adapt to their changing environments. They can be, but are not necessarily omnivorous by definition.

Such animals usually eat plants in the summertime, but often hunt other animals in winter. The defining trait of euryphages is that they are adapted to a continuous change in their ration (while other animals, carnivores, herbivores, and omnivores alike might only rely on a few, unchanging sources of food, like a koala eating only specific type of eucalyptus all year round). The enzymatic spectrum of euryphages as well as omnivores is likely to include enzymes characteristic of both carnivores and herbivores

Anatomical features common to omnivores include:

FLAT MOLARS AND SHARP TEETH developed for some grinding and some tearing.Omnivores tend to have all the different types of teeth. Good examples of omnivores are raccoons, bears, and humans.

EYES. Although they can have eyes on the sides of their heads, like those of prey animals, more often their eyes are towards the front of their heads, like those of predators.

MEDIUM LENGTH DIGESTIVE TRACTS that provide the flexibility to digest both vegetation and animal proteins.

SALIVA CONTAINS CARBOHYDRATE DIGESTING ENZYME AMYLASE which is responsible for the majority of starch digestion. The defining and most enzymatically-demanding feature of omnivorous animals is that with food, they receive protein, fats, and carbohydrates altogether. We have reviewed above the different types and specific enzyme examples that act in breaking down specific nutrients, along with the final products of their mediated breakdown. Omnivorous animals are likely to have representatives of all these enzyme types in their enzyme arsenal.

COMPUTER SCIENCE

Solutions must be typed and submitted in one of following formats:

.txt .c .cpp .java .py

Solutions written in Java, C, C++, Python and pseudo-code are accepted.

Pseudo-code guidelines are at

http://users.csc.calpoly.edu/~jdalbey/SWE/pdl_std.html

5 points:

Define a procedure that takes a list of numbers as input and returns the sum of the squares of the input values that are even.
(Hint: You can test whether a number is even by seeing if the number mod 2 is 0, that is, x % 2 == 0)

Solution:

```
def evenSquares(someList):
        return sum([x**2 for x in someList if x%2 == 0])
```

10 points:

An astronaut is on a cylindrical spaceship with M airlocks (exits) positioned around in a circle. He needs to exit one of the airlocks, go all the way around in a circle, and come back through the same airlock. He has a limited capacity oxygen tank: a completely filled tank is only enough to last him 600 seconds. There is an oxygen source at each airlock where the astronaut can re-fill his tank.

You are given:

n = the number of airlocks, numbered 0 to n-1.

job[n] = an array of size n, where job[2] represents the time it takes from the astronaut to get from airlock 2 to airlock 3. If there are 6 airlocks, then job[5] = time to go from airlock 5 to airlock 0.

oxygen[n] = the number of seconds of oxygen available for refill at airlock [n]. Upon arriving to airlock n, the astronaut can re-fill his tank, but his tank still cannot exceed the 600 second capacity.

Task:
Determine whether the astronaut can complete his task without having to get back into the spaceship to re-fill his tank. That is, is there an airlock that he can exit through and go in either direction so that he can make it all the way around and re-enter through the same airlock without running out of oxygen? If it is impossible, output “no”. If it is possible, output the number of an airlock that he can exit.

Example 1:
Input:
n = 4
job[] = [700, 100, 100, 100]
oxygen[] = [100, 100, 500, 500]

Output: “No”. (This is because job[0] = 700, which is greater than the tank’s capacity.)

Example 2
Input:
n = 4
job[] = [100, 500, 100, 100]
oxygen[] = [100, 0, 100,0]

Output: “Yes”. The astronaut starts at airlock 1, uses up 500 of his 600 of oxygen, refills back to 200 at n=2, uses up 100, left with 100, cannot refill at n=3, uses up 100, left with 0, refills to 100 at n = 0, uses up 100 to get to n = 1, and gets back into the spaceship with no oxygen left.

Solution:

```
capacity = 600

For each m representing the ID of the airlock (0 =< m < M):
        fuel_left = 500
        i = 0
        valid_trip = true
        while i is not greater than M:
                //re-filling up tank at each station:
                fuel_left = the smaller of: (fuel_left + oxygen[i+m modulo M], capacity)
                //moving to the next station
                fuel_left -= job[i+m modulo M]
                if fuel_left < 0:
                        valid_trip = false
                        exit the while loop.
                i ++;

      if valid_trip is true:
               Return m
Return no
```