

**PROBLEM OF THE
MONTH**



October, 2019

MATHEMATICS

5 points: Find all prime numbers p such that for each of them there exists a natural number n so that $\sqrt{n} + \sqrt{n+p}$ is a natural number.

Hint: Both \sqrt{n} and $\sqrt{n+p}$ have to be natural numbers for their sum to be a natural number, and the only way that is possible is if n and $n + p$ are squares.

10 points:

- 1 point: Find a solution to the equation $x^2 - xy - y^2 = 1$ where x and y are natural numbers. (Please write it as an ordered pair (x,y))
- 7 points: Find all pairs of natural numbers (x,y) such that $x^2 - xy - y^2 = 1$.
- 2 points: Prove that there are no other possible solutions.

Hint: Once you have found the pattern and some pair of numbers that works, you can use mathematical induction to prove that this pattern always works.

PHYSICS

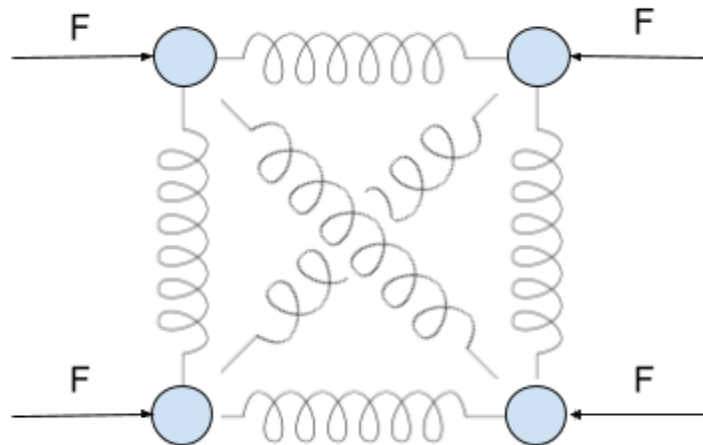
5 points:

When at the restaurant chopsticks come out of the dishwasher, they are not aligned -- roughly half the chopsticks have tips pointing one way, and half the other way. Propose a method to efficiently sort them based on their alignment. Explain the physical mechanism, and perform an experimental demonstration of your method. You can use (same-length) sharpened pencils to model chopsticks.

Hint: think about a single chopstick balancing on the edge of a table (perpendicular to that edge).

10 points:

It is known that materials tend to expand in directions perpendicular to the direction of compression. To model this, consider a cell in which interatomic bonds are represented by springs as shown in the Figure below. The spring constants of all the springs are the same. Find the ratio of the expansion of the cell in the perpendicular (vertical) direction to its compression along the direction of the applied force (horizontal). Assume all the deformations to be small.



Hint: Consider the balance of forces acting on the “atom” in the upper left corner.

CHEMISTRY

5 points:

During Sigma2019, the Mystery of chemical glassware semilab included the experiment called "extraction of caffeine from coffee": coffee was extracted using an organic solvent called ethyl acetate, the organic phase is separated and evaporated, thereby yielding solid caffeine. Imagine that Mark forgot to bring ethyl acetate to Sigma. Would it still be possible to do that experiment if the only organic solvents available to Mark were: ethanol, hexane, acetone, dichloromethane, acetic acid, and all needed inorganic chemicals? What should Mark do in that situation?

Hint:

This type problems can be solved using two different approaches: we either prepare a solvent that we need using available chemicals, or we use another solvent with similar properties. Which approach is the best in our case? The first one? The second one? Maybe, both?

10 points:

Imagine you have to prepare as much hydrogen as possible, and you can spend \$40 for chemicals (you can buy whatever you want from Amazon; shipping cost is not included). What is the maximal volume of hydrogen that you can prepare? Shipping cost is not included.

Hint:

Obviously, one of the simplest ways is to use some metal that produces hydrogen in a reaction with some other chemical. The difference between this problem and similar problems of that kind is that you have to calculate the amount of hydrogen per \$40, not the amount of hydrogen per one mole (or gram, etc) of chemicals.

BIOLOGY

5 points:

Mark has a dog, her name is Marta (on the right photo). She, like all laikas, is a passionate hunter. When she and Mark are walking in the woods, she loves to hunt chipmunks, and the hunt usually happens like this: Marta walks along the path; when the chipmunk sees her, it squeaks loudly and hides in the hole. Marta runs up to the hole, quickly digs it up (on the left photo), and, if Mark does not pull her away, she catches the chipmunk. It is clear that had the chipmunk not squeaked, Marta probably would not have noticed it. Obviously, chipmunk's squeaking is some instinct. Why is it needed?



Hint:

All chipmunks behave similarly, which means that is some inherited instinctive behaviour. That means that trait is beneficial for survival, although in this situation it leads to chipmunk's death in almost 100% cases (of course, we mean a hypothetical situation when Mark doesn't stop Marta). That seemingly obvious contradiction with the Darwin's "survival of the fittest" principle can be easily resolved if we remember that one word is missing. What exactly did Darwin meant under "survival of the fittest"?

10 points:

In the science fiction novel "The Gods Themselves" by Isaac Asimov, the author describes some alien race with three sexes ("rational", "emotional", and "parental"). He explains the mechanism of their reproduction (all three sexes participate in reproduction, and always give birth to one "rational", one "emotional", and one "parental" child). Asimov tells nothing about the genomic organization of that alien species, but let's imagine that it is based on some nucleic

acid, which is similar to our DNA, and inheritance rules are similar. Please explain how can chromosomes of such species be organized to produce three different sexes, as a result of sexual reproduction.

Hint:

Of course, the species described by Azimov are imaginary, so we can just speculate about their genome organization. However, we should keep in mind that the genome organization should look reasonable, and each of three sexes is supposed to have a chromosome set that will reliably lead to no loss of genetic information, and the ratio between the three sexes in the next generation must be exactly 1:1:1. Theoretically, a diploid genome can provide that. How exactly?

COMPUTER SCIENCE

- You can write and compile your code here:
<http://www.tutorialspoint.com/codingground.htm>
- Your program should be written in Java or Python
- No GUI should be used in your program: eg., easygui in Python. All problems in POM require only text input and output. GUI usage complicates solution validation, for which we are also using *codingground* site. Solutions with GUI will have points deducted or won't receive any points at all.
- Please make sure that the code compiles and runs on
<http://www.tutorialspoint.com/codingground.htm> before submitting it.
- Any input data specified in the problem should be supplied as user input, not hard-coded into the text of the program.
- Submit the problem in a plain text file, such as .txt, .dat, etc.
No .pdf, .doc, .docx, etc!

Credit card companies tend to collect all purchases made by a particular client in a given month and mail a report to the client's home in a batch known as the "monthly statement". The clients then pay for their purchases with cash from their bank accounts.

As an incentive for customers to use their credit cards, there is also a reward mechanism: clients earn "points" on all/most purchases they make using their credit cards, typically in proportion to the dollar amount of the purchases. The points can then be redeemed and used to "cover"/pay for some of the purchases in lieu of using cash. We will assume that i) all available points are to be redeemed at once for a combination of charges and that ii) no partial covering of charges is allowed: a particular charge is either fully covered or not covered at all with the points. Any number of points that are left without covering charges in full will disappear and are effectively wasted.

5 points:

Write a program that takes as input a list of positive integer valued (not necessarily sorted in any manner) charges and finds out if there exists two pairs of charges that have the same total amount. If there exists a list of 4 separate charges a , b , c , and d , such that $a+b=c+d$, your program should print "YES" and list the charges. Otherwise the program should print "NO". Assume that charges are all positive integers. If the input contains less than 4 elements or if the input cannot be interpreted as integers, the program should indicate so.

Hint:

It's convenient to use a map of all sums and a tuple of their constituents. Alternatively, you can use an array and sort it.

10 points:

Write a program that takes as input a list of positive integer valued charges and a positive integer point value earned, and outputs the combination of charges that wastes the least number of points when redeemed and the actual point value wasted.

For example, input of charges [3, 4, 1, 9, 100, 4, 1, 6, 1] and point value of 111 would result in the output of charges [1, 1, 9, 100] as the best combination with the wasted point value equal to $111 - (1 + 1 + 9 + 100) = 0$. Input of charges [3, 4, 1, 9, 100] and point value of 106 would result in the output of charges [1, 4, 100] with $106 - (1 + 4 + 100) = 1$ point wasted.

In case the inputs are non-numerical/non-integer/non-positive, or if not a single charge can be covered by the point value provided, then the program should indicate so.

If more than one combination of charges results in the same minimum amount of points wasted (there is a tie), it suffices to output only one of the combinations along with the points wasted.

Hint:

Use principles of dynamic programming: consider increasing in size sets of charges and remember accumulated totals in a table as you go.

LINGUISTICS

5 points:

Consider the following words from a North-East Caucasian language. Apart from translations, you are given the sample sentences where the particular form would be used.

	Gloss	Sample sentence
<i>q'atluvu</i>	in the house	I'm in the house
<i>q'atluxux</i>	behind the house	I pass behind the house
<i>q'atluvatu</i>	from inside the house	I go from inside the house
<i>q'atlulu</i>	under the house	I am under the house
<i>q'atluj</i>	on top of the house	I am on top of the house
<i>q'atluvun</i>	into the house	I enter into the house
<i>q'atluxatu</i>	from behind the house	I come from behind the house
<i>q'atlulun</i>	under the house	I go under the house
<i>q'atlujx</i>	over the house	I walk over the house (over the roof)

Translate the following phrases (as used in the corresponding sentences) into this language and explain your reasoning.

1. from under the house I exit from under the house
2. through the house I pass through the house
3. on top of the house I enter on top of the house (its roof)

10 points:

Consider the following sentences from a North-East Caucasian language and their translations:

1. dis hibat:ur došdur di I have a good sister
2. wel x:allu ušdu wak:urši wi We see a bad brother
3. bolo x:allub xoŋn bi We have a bad cow
4. ez jamut q'onq' erxinši i I forget this book
5. bez jamub c'ai bokorši bi I hear this goat
6. el hibat:ut aqx' ak:urši i We see good meat

Translate into this language:

1. We hear this sister.
2. We have a bad book.
3. I forget a good brother.
4. I have a good goat.