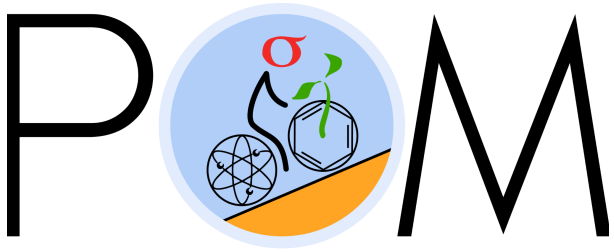
PROBLEM OF THE MONTH



October, 2021

2021 - 2022

MATHEMATICS

5 points:

How many distinct (non-congruent) rectangles with integer sides have area equal to 2048?

10 points:

How many distinct (non-congruent) rectangular cuboids with integer sides have volume equal to 2048?

PHYSICS

5 points:

You are standing on a skateboard (total mass M), and have a handful of marbles in your pocket. Total mass of the marbles is m, and $m << M$. You throw one marble at a time horizontally, in one direction, with speed v. How fast will you be going once you are out of marbles?

10 points:

It turns out that light carries momentum! If a light beam has energy E, then the magnitude of its momentum p is given by $E = cp$, where c is the speed of light.

Hypothetical methods of transport known as 'solar sails' take advantage of this, similar to how sails of sea ships take advantage of the momentum carried by wind.

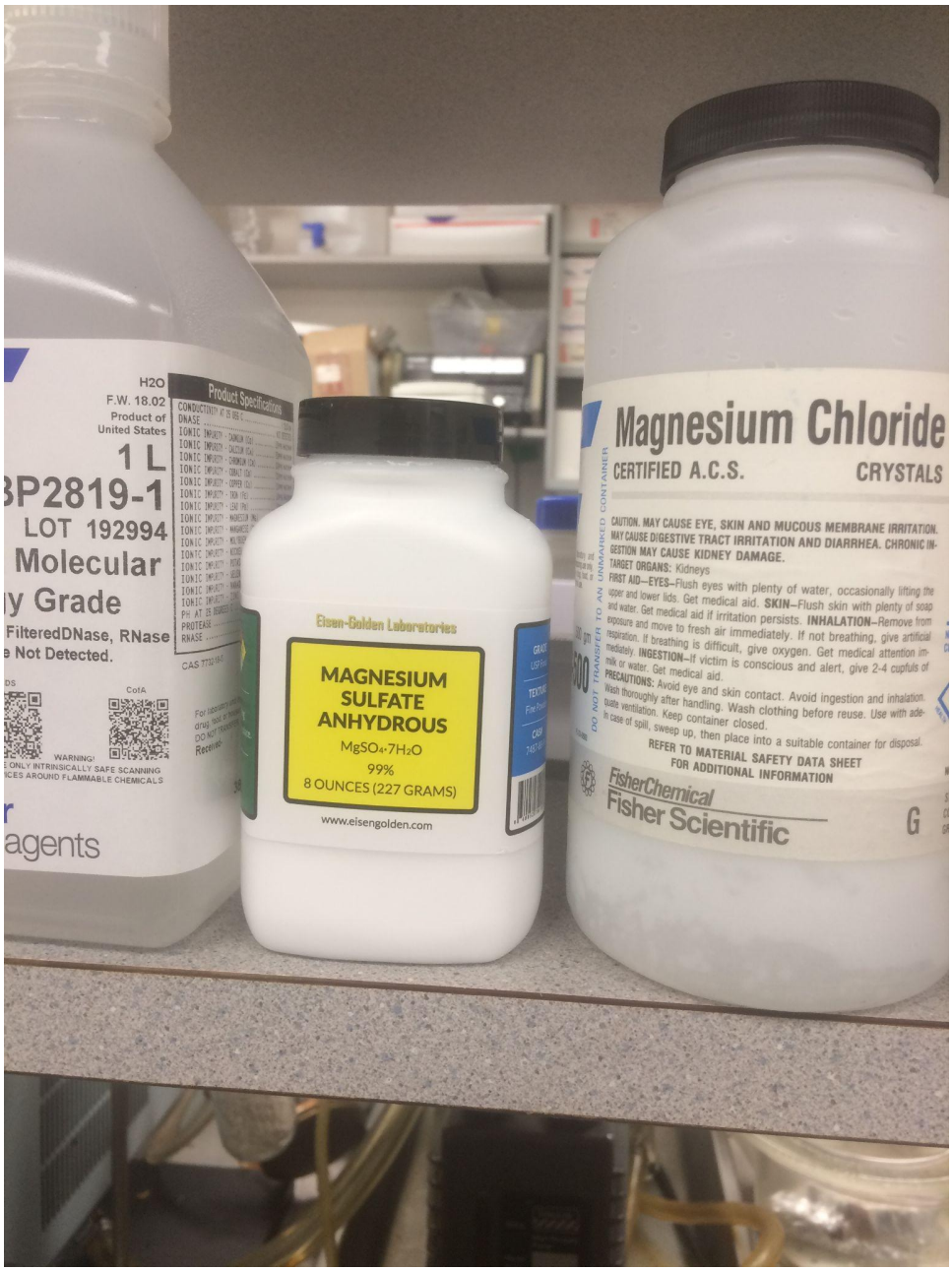
Suppose a solar sail of area A and mass m is orbiting around the sun at radius R. What acceleration does it experience due to the light from the sun, if the sail is oriented to face the incoming light? Assume the sun emits total power P. Note that the solar sail is made of material that absorbs all light.

How will your acceleration change if the solar sail is made of a reflective material?

CHEMISTRY

5 points:

The photo (see below) shows a shelf in Mark's lab. Each bottle contains chemicals purchased from certified suppliers. Mark took this photo because some of the labels cause what is commonly referred to as "cognitive dissonance". Explain what is wrong with the label, and propose the experiment that would allow Mark to resolve the problem.



10 points:

I cannot guarantee that this story was true, but rumor has it that among the papers left over from Sir Conan Doyle there was a draft of one story tentatively titled “Strange Crystal”. This story tells of a very rare and very valuable mineral stolen from a wealthy Mr Penrose, who earned his capital collecting and selling various unusual natural objects and artifacts. One of his most valuable items was stolen from his collection: a very rare "Patagonian sapphire", a dark blue natural mineral that, according to Penrose, was found in some remote cave in Patagonia. The Patagonian sapphire was unique because of its shape: it was a prism, and its cross-section was a regular pentagon. Penrose was the only person in the world to own a Patagonian sapphire, which was the most valuable item in his collection. Unsurprisingly, Penrose insured his Patagonian sapphire at Lloyd for a fantastic £ 10 million. Shortly thereafter, the crystal was stolen from the house of Penrose. Lloyd's representative came to Holmes and asked if he could find this extremely valuable item. Hearing about the unusual form of the mineral, Holmes immediately concluded that Lloyd was not obliged to pay Penrose money and might even sue Penrose for fraud because the Patagonian Sapphire was a clear fake, and the alleged robbery was staged.

Why did Holmes come to this conclusion?

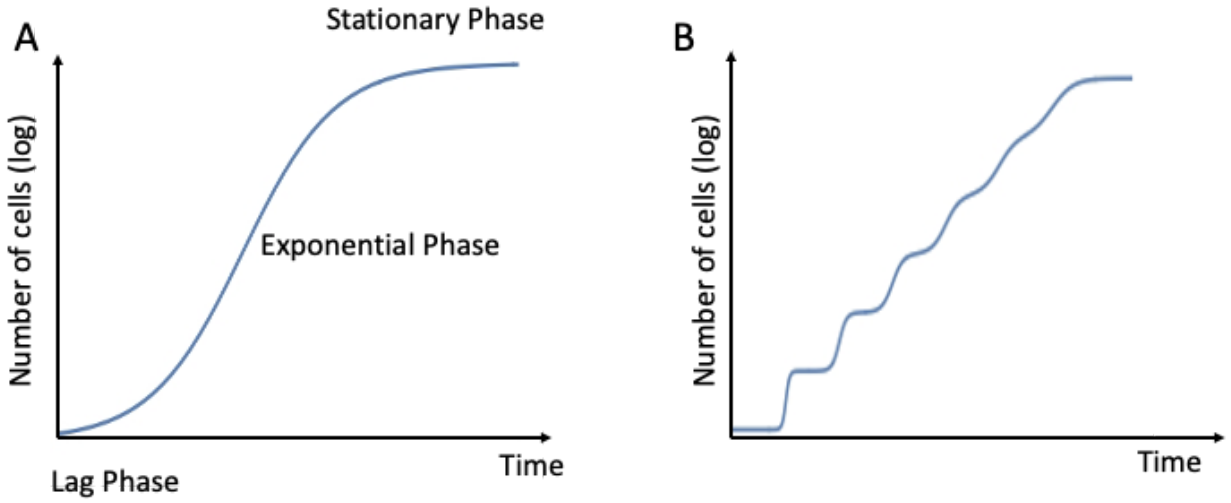
If Holmes lived in 2010, could he be equally categorical?

BIOLOGY

5 points:

Usually, the growth of a bacterial culture occurs as shown in the Fig A (see below): after the initial lag phase (when the growth of bacteria is slow), bacteria begin to grow exponentially (i.e., their growth rate is proportional to the density of the culture, so it doubles after the same time interval). This phase looks like a straight line in the left figure because the Y-axis is on a logarithmic scale, which means it increases as “1, 10, 100, 1000…” instead of our usual “1, 2, 3, 4…”. (This is a standard way of plotting the growth of bacterial cultures, as well as other processes where the amplitude of the change in numbers is extremely large.) As a rule, the growth of the overwhelming majority of bacterial cultures, including *E. coli*, one of the most popular bacteria in research laboratories, obeys that law.

However, if you take a small sample of the *E. coli* culture, incubate it at an elevated temperature (45 degrees), add it to a fresh media and place it under optimal temperature conditions, the growth curve will be as shown in the Fig. B. Explain why this is happening. What else can cause bacteria to grow, as shown in Figure B?



A B

10 points:

During the 19th and first half of the 20th century, scientists believed that all species belong to two domains of life: eukaryotes and prokaryotes (a. k. a. bacteria). Recently, scientists have found that one group of unicellular organisms, despite their visible similarity to bacteria, represent a completely different domain of life, which is now called "archaea".

Archaea are very unusual organisms that look like bacteria, but differ from them ... in almost everything. Their proteins are completely different (and they are more like our own, eukaryotic proteins), their cell membrane is different, their genomic organization is different, their

metabolism is different. Archaea can be found everywhere, even in such unfriendly places as Antarctic lakes, Yellowstone geysers, underwater volcanoes, and even nuclear reactors. Some of them live in our guts (and they are responsible for producing methane) and similar places. However, until now, no archaea species has been identified as the primary cause of any human disease.

Propose possible explanations for that phenomenon.

LINGUISTICS

5 points:

The phrases below are from the language of an Indigenous People with the translation in a random order.

ava hoveny iyuuk
ahaaly uwaak
avaaly uwaak
masahay ahvay aym
masahay ahvay hoveny ičook

I am in the house
I gave a dress to the girl
He is by the water
I saw that house
I made that dress for the girl

1. Match the sentence to the correct translation. Explain your reasoning.
2. Translate: *He made that house for the girl*. Explain your reasoning.

10 points:

Below are some phrases from Cartínese — a language that will evolve several hundred years in the future on the islands of Cartí in Panama. Translations are presented alongside each sentence, in scrambled order.

(For phonetic reading: c = /ts/, q = /tʃ/, x = /ʃ/)

A. cui ye lulo na be qewana qe ho moha
B. cui qisin dei ese a ho qewanay
C. we a qe be suruy
D. ye qisin dei cui we be lemuy so
E. we ye qisin dei a be xanay
F. cui we ye qewan la be rakay ra
G. ye lemo dei ye su ho xana
H. cui ye lekera la qe be xanaha so
I. ye lulo qe ho xanay koi
J. ese su ho surun koi
K. cui su a ho bodanan koi
L. wocasa dei a xume be bodana ra
M. cui we a be bodanay ka qisin

1. he doesn't like himself
2. our dogs play among themselves
3. we didn't see the doctor!
4. she's going to set me up
5. your dog likes to play
6. the doctor said that (his patient) is sick
7. they call each other dogs
8. she sees her dog
9. you all get called by each other
10. we don't have to play ball
11. i might call my lawyer
12. you are all likeable
13. i see your ball

N. we ye be rakay ra
O. qisin dei ye su ho suruy ka qewana
P. lekera mu ye we be burunay ho danay so

14. the ball isn't visible
15. they will set up the game!
16. their dog bit them

1. Match each sentence to the right to a sentence to the left.
2. One sentence on the left is written incorrectly; rewrite the sentence to reflect the meaning of the sentence to the right, and explain why.
3. Translate the following sentences into Cartínese:
    a. I like that you aren't sick.
    b. You all have eyes!
    c. She will call herself.
    d. They are set-uppable.

COMPUTER SCIENCE

- Your program should be written in Java or Python-3
- No GUI should be used in your program: eg., easygui in Python
- All the input and output should be via files named as specified in the problem statement
- Java programs should be submitted in a file with extension .java; Python-3 programs should be submitted in a file with extension .py
- **No .pdf, .doc, .docx, etc! Programs submitted in incorrect format will not receive any points!**

5 points:

There are *n* building projects currently proposed at sites arranged in a row along the main commercial street in Elizaville, NY. Each one, if building there were allowed, would bring a specific amount of revenue to the town: site 1 would bring revenue r_1 (in dollars), site 2 would bring revenue r_2, etc.

The town would like to approve *three* of these projects to bring in as much revenue as possible. However, there is a density constraint: no two approved projects can be next to each other! So their goal is to pick three projects i, j, k which maximize the sum of the revenues $r_i + r_j + r_k$, but where $j \geq i + 2$ and $k \geq j + 2$.

Write a program that takes as input the revenues $r_1, \ldots, r_n$ (in a file named **input.txt**, formatted as one integer per line) and outputs the list of projects to approve (into an output file named **output.txt**, also one integer per line).

For example, if input.txt contains:

```
8
10
12
11
2
4
5
```

you would select projects 2, 4, 7, for a total revenue of 10 + 11 + 5 = 26, and the output.txt file would contain:

```
2
4
7
```

Your program should run in a few seconds when the input has length up to *n* = 50.

10 points:

You were so successful in helping out Elizaville, NY that you've been asked to help out a much larger city—Los Angeles, CA—with a similar problem. Again, there are *n* sites arranged along a street, Figueroa Boulevard. Figueroa is about 30 miles long and has space for lots of building sites, so *n* could be huge.

This time, however, there is no limit on the number of sites that may be selected. The only constraint is that you may not select two sites that are next to each other.

Write a program that takes as input the revenues $r_1,\ldots,r_n$ (in a file named **input.txt**, formatted as one integer per line) and outputs the list of projects to approve (into an output file named **output.txt**, also one integer per line).

For example, if input.txt contains (same as above):

```
8
10
12
11
2
4
5
```

you would select projects 1, 3, 5, and 7, for a total revenue of 8 + 12 + 2 + 5 = 27, and the output.txt file would contain:

```
1
3
5
7
```

For this problem, your program should be able to handle larger inputs—with *n* as large as 10,000—in under a second. We recommend you design an algorithm that solves the problem in time roughly proportional to *n*.