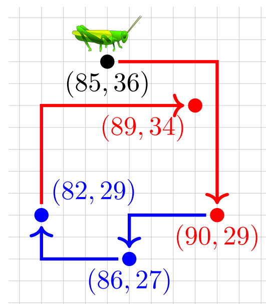
SigmaCamp's *Problem of the Month* Contest

SEPTEMBER 2023

Mathematics

5 points:

Gerry the grasshopper lives on an infinite grid with integer coordinates. He starts at (85, 36), and his home is at (0,0). Each hop he takes, he can either go 5 in one direction and 7 in another, or 4 in one direction and 2 in another. Can he get home?

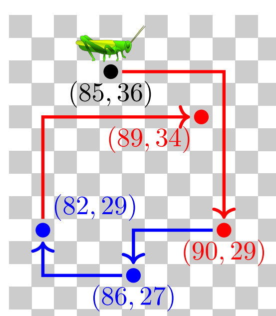


Hint:

Imagine that Gerry lives on a checkerboard.

Solution:

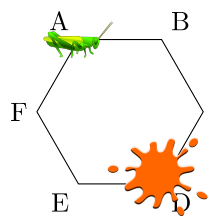
It is unfortunately impossible for Gerry to get home. The sum of his initial coordinates is $85 + 36 = 121$, which is odd. Every time Gerry hops, the sum of his coordinates changes by 2, 6, or 12 (4 ± 2 or 5 ± 7). Since these are all even, the sum of Gerry's coordinates will remain odd. Since the sum of his home's coordinates is even ($0 + 0 = 0$), he will never be able to get home.



Another way to think about it is to imagine that Gerry lives on a checkerboard. If he starts off on, say, a black square, he will only ever be able to visit other black squares. Since (0,0) will unfortunately end up being white, Gerry will never be able to get there.

10 points:

Gerry is now jumping on vertices of a regular hexagon. At each step, he jumps to one of the two neighboring vertices, choosing each possibility with probability 1/2. Gerry started at vertex A. There is a pool of orange paint at vertex D. If Gerry jumps there, he will be stained with orange paint. What is the probability that he won't be stained after 25 jumps?

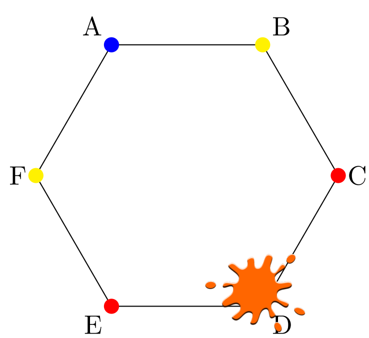


Hint:

Are there some vertices we can consider equivalent?

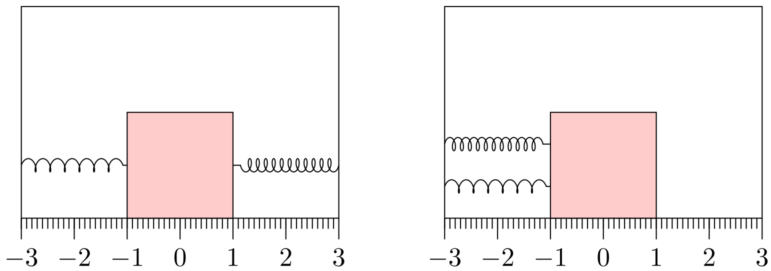
Solution:

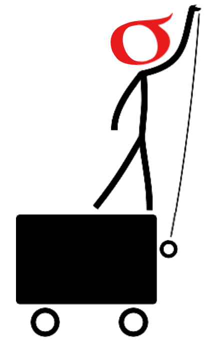
Color vertex A blue, B and F yellow, and C and E red. After the first hop, Gerry will be at a yellow vertex. After that, with probability $\frac{1}{4}$, he will end up at vertex D in two hops. With the remaining probability $1 - \frac{1}{4} = \frac{3}{4}$, he will end up back at a yellow vertex. If the grasshopper manages to avoid vertex D, the same thing will happen for every subsequent pair of hops. Since there are $24/2 = 12$ pairs of hops remaining, the probability Gerry does not get stained by the paint is $(\frac{3}{4})^{12}$.



Physics

5 points:





Two identical cubes of mass $m = 0.03$ kg are placed in identical frictionless boxes and attached to the boxes via Hookean springs. One cube is attached on the left and right side by springs with spring constants $k_1 = 130$ and $k_2 = 70$ N/m, respectively. The other is attached by two springs on the left side, which also have spring constants k_1 and k_2 N/m. The boxes are then rotated 90° counterclockwise. What distance are the cubes from their starting positions when they come to rest? (Assume that neither spring hits the side/bottom of the box.) You may use $g = 10$ m/s^2.

Notes:

1. No points will be given for an answer without a justification.
2. Remember, if you are unfamiliar with terms like "Hookean", you are allowed to Google or look at education resources like Khan Academy. Just remember to cite your sources!

Hint:

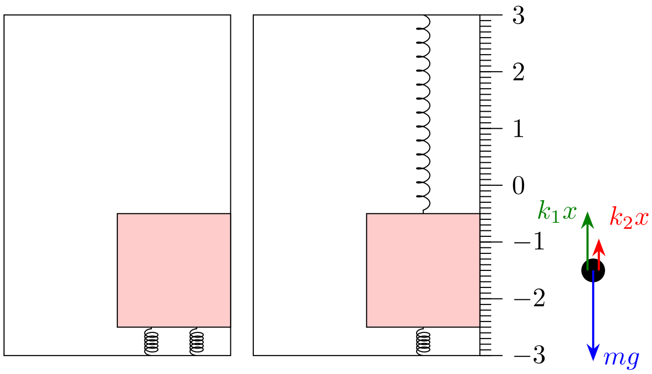
No hint this month.

Solution:

Answer: 0.015 m

We will consider the right block first. Hooke's Law states that the force from a spring is $-kx$, where x is the displacement and k is the spring constant. The force from both springs will be opposite the direction of the displacement (i.e. if the block is moved to the right, both springs will pull to the left). The forces add, so the total force from the springs $F_s(x) = -k_1x - k_2x = 200\text{N/m} \cdot x$.

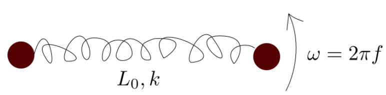
The force on the left block will from each spring will be the same. Although whether the springs are stretched or compressed will change, they will both still pull/push the box back toward the original equilibrium position. This means that the total force from the springs will still be $F_s(x) = -k_1x - k_2x = 200\text{N/m} \cdot x$. Since gravity, the only other force considered, will affect both boxes equally (they have the same mass), both boxes will fall the same distance.



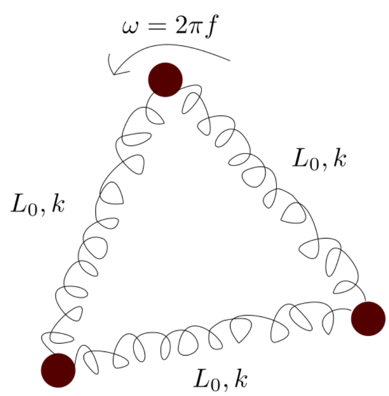
In order for the cubes to be at rest, the total force on them must be 0. Then $F_s(x) - F_g = 0$. Substituting, $200\text{N/m} \cdot x - mg = 0$. Solving for x, we find that the distance at which the forces balance and the block will come to rest is $\frac{mg}{200\text{N/m}} = \frac{0.03\text{kg}\cdot 10\text{m/s}^2}{200\text{N/m}} = 0.015$ m.

10 points:

(a) Two masses of mass m are connected by a spring of rest length L_0 and spring constant k. The entire construction spins f times per second around its center of mass, with angular frequency $\omega = 2\pi f$. What is the radius of the orbit that the masses follow?



(b) This time, *three* masses of mass m are connected by springs, as shown. The springs all still have spring constant k and rest length L_0. Will the radius of orbit of the masses decrease or increase with respect to the previous case?



Hint:

No hint this month.

Solution:

Answer:

$$\textbf{Part (a):} \quad R = \frac{kL_0}{2k - m\omega^2}, \quad \textbf{Part(b)}: R = \frac{\sqrt{3}kL_0}{3k - m\omega^2}. \tag{1}$$

(a) Two equations are important – the restoring force of a spring, and the centripetal acceleration of an object undergoing circular motion. A spring with spring constant k and equilibrium length L_0 will exert a force

$$F = k(L - L_0) =: k\Delta L \tag{2}$$

when stretched to a length $L = L_0 + \Delta L$. An object undergoing circular motion at radius R and frequency ω will have a centripetal acceleration of

$$a = \omega^2 R. \tag{3}$$

When the spring is stretched to length L, the two masses are orbiting at radius $R = L/2$. We must therefore relate the force on the mass and its acceleration as

$$F = ma \quad \Longrightarrow \quad k(2R - L_0) = m\omega^2 R \quad \Longrightarrow \quad R = \frac{kL_0}{2k - m\omega^2}. \tag{4}$$

It is instructive to analyze two limits of this relation. First, as the frequency ω goes to 0, the diameter of the circular motion $2R$ simply approaches the rest length of the spring, L_0. This makes sense, as if the masses are sitting there and not moving the spring will remain at its rest length.

The second (and perhaps more surprising) observation as that the radius of orbit approaches infinity as $\omega^2 \to 2k/m$. This is a sign that the model of an ideal spring with linear restoring force must start breaking down at this frequency, and either more powerful restoring forces (proportional to $(\Delta L)^2$) must come into play or the spring simply rips apart.

(b) The method for solving this problem is identical, we must simply do a little bit more geometry for deducing the lengths of the springs when the masses are orbiting at radius R. An equilateral triangle of side length L is inscribed in a circle of radius

$$R = \frac{1}{\sqrt{3}} L. \tag{5}$$

The restoring force (pointed along the radius of the circle) is at angle $\pi/6$ with respect to the spring, and there are two springs attached to each mass, so the centripetal force is

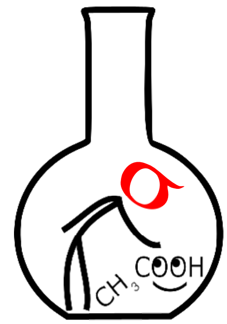
$$F = 2k(L - L_0)\cos\pi/6 = 3kR - \sqrt{3}kL_0 = m\omega^2 R. \tag{6}$$

Putting everything together, as before, we find

$$R = \frac{\sqrt{3}kL_0}{3k - m\omega^2}. \tag{7}$$

Compared to the case of two masses and a single spring, the masses orbit at a larger radius. The expression now breaks down at a slightly larger frequency frequency $\omega^2 = 3k/m$.

Chemistry



5 points:

Oh no! You didn't follow standard laboratory procedure and forgot to label four containers, each containing a different white powder. The four possible compounds are:

- Sodium carbonate
- Cupric sulfate
- Citric acid
- Lithium aluminum hydride

Since they are visually indistinguishable, you don't know which is which. Aside from these compounds, you also have access to plenty of beakers, as well as the following chemicals:

1. Hexane
2. Nitromethane
3. Benzene
4. Distilled water
5. Absolute ethanol
6. Carbon tetrachloride

Which one chemical from the list can you use to determine the identity of each of the four compounds, and how?

Hint:

Although it is not absolutely necessary, but you may use the following trick: after you have identified some of those chemicals, you may use them as reactants to identify remaining compounds.

Solution:

Answer: Distilled water

Using distilled water, you can immediately recognize copper [1] sulfate and $LiAlH_4$. The former immediately becomes blue, because four water molecules tightly bind to the Cu^{2+} ion, thereby changing its optical properties; the latter reacts violently with water:

$$LiAlH_4 + 4\,H_2O \longrightarrow LiOH + Al(OH)_3 + 2\,H_2$$

The other two compounds just dissolve in water. Of course, it is possible to confirm that *one of them* is sodium carbonate, because if you mix these two solutions together, bubbles of CO_2 form.[2] However, the point is that nobody can prevent you from using already identified compounds as reagents: you *already have cupric sulfate, and you can use it as an auxiliary chemical.* By adding $CuSO_4$ to each of those two

[1] a.k.a. cupric, for the salts of the metals that exists in two oxidative states are called "-ous" if the metal is in a lower oxidative state and "-ic" is the metal is in a high oxidative state: thus, Cu_2O is "cuprous oxide", whereas CuO is "cupric oxide".

[2] You may try to write the equation by yourself.

solutions, you immediately see which is which: there is no change when cupric sulfate is added to the citric acid solution, but a sodium carbonate solution produces a copious precipitate of $CuCO_3$.[3]

$CuSO_4 + Na_2CO_3 \longrightarrow CuCO_3 + Na_2SO_4$

10 points:

In 2023, our Sigma faculty members, Lena and Natasha, were preparing for a marine cruise. Part of the cruise research program was to measure the uptake of inorganic carbon by phytoplankton in an anoxic zone. To measure the absorption rate, the researchers needed a stock solution of labeled sodium bicarbonate ($NaH^{13}CO_3$). Lena prepared 1.1M sodium bicarbonate solution and 6.0M sodium chloride solution. According to the experimental protocol, these two solutions were supposed to be mixed directly in a syringe in a 1:1 ratio immediately before the experiment and quickly injected into a sample of ocean water taken from a depth of 500 meters. Something went wrong, however: immediately after mixing, the two solutions became cloudy, and the resulting precipitate clogged the syringe needle.

(a) Determine what compound the precipitate is.

(b) Explain why the precipitate formed.

(c) Propose a modified protocol that allows preparation of the final solution with a required concentration of the bicarbonate ion ($[H^{13}CO_3^-] = 0.55$M) and a maximal possible salinity.

Hint:

Read about a "solubility product". Using this concept, it will be easy to understand the mistake made by Lena.

Solution:

Answer: the precipitate is sodium bicarbonate.

To understand why it precipitates, we need to understand the solubility product concept. First, both sodium bicarbonate and sodium chloride are salts (a.k.a. ionic compounds), which means that during dissolution they dissociate: there is no sodium bicarbonate or sodium chloride in solution, just sodium ions, chloride ions, and bicarbonate ions.

Second, the solubility of each salt is defined by its solubility product. Consider a salt with a formula AB, which dissociates upon dissolution to produce cations A and anions B. To denote actual concentration of ions, chemists use square brackets. In a saturated solution of AB, concentrations of each ions ($[A^+]$ and $[B^-]$, accordingly) reach a maximal magnitude, and their product is called a "solubility product" (SP).

$$SP = [A^+][B^-] \tag{8}$$

Importantly, SP is a constant, and it depends only on temperature. But $[A^+]$ and $[B^-]$ don't have to be constant: you may, e.g., increase $[A^+]$, and that will lead to a decrease of $[B^-]$. How can you do that? You may, for example, add some other salt, e.g. AC. Like AB, it dissociates, thereby increasing concentration of A^+. As a result, $[B^-]$ will go down. How can it happen? For example, by precipitation of some AB.

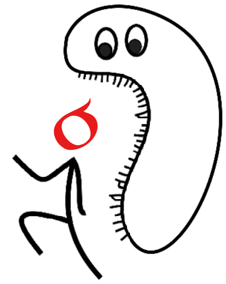
If a solution of a salt with a formula AB is almost saturated, and you add another salt, AC, where one ion (A) is the same as in AB, [A] increases, so the product of [A] and [B] may become bigger than the solubility product of AB. That will lead to precipitation of some amount of AB.

[3]Actually, the precipitate is so called *basic copper carbonate*, of $Cu_2(OH)_2CO_3$, or a mixture of copper hydroxide and copper carbonate, but that nuance is hardy important in our case.

Now imagine that AB is sodium bicarbonate and AC is sodium chloride. The sodium bicarbonate solution is nearly saturated, so when you add sodium chloride, a total concentration of $[Na^+]$ becomes so high that the product $[Na^+][HCO_3{}^-]$ becomes greater than sodium bicarbonate's SP. As a result, an excess of $NaHCO_3$ precipitates.

To fix the problem with precipitation, less concentrated NaCl solution should be used. Unfortunately, it was impossible to replace NaCl with KCl (potassium ions were undesirable for that experiment).

Biology

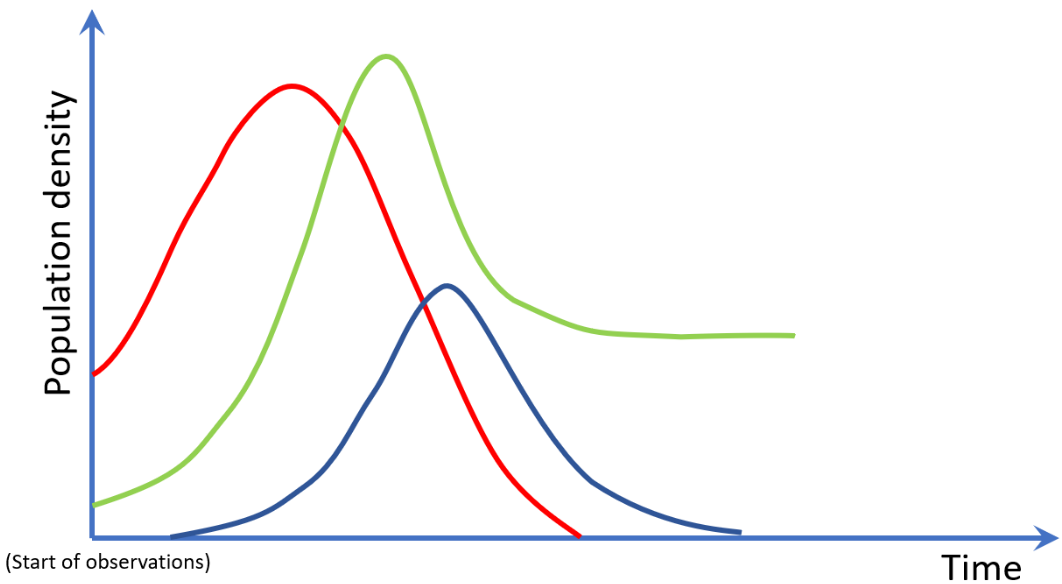


Welcome to the Biology POM 2023-2024! We're very excited to introduce you to the amazing world of Biology through some fun and thought provoking problems this year. When answering questions, we expect you to do research and consult outside sources, but you do need to cite every source you use in a consistent format (MLA 9th Edition). Here are some resources to help you with that! As always, don't hesitate to contact Sanjana Rao (sanjanarao@uchicago.edu) or Jacob Kudria (jacob@kudria.net), this year's Biology subject leads, if you have any questions.

5 points:

For several years, naturalists have been studying how the population of three invasive beetle species changes on a small tropical island. All three species were accidentally introduced at different times from a neighboring island. The changes in population density of the three species are summarized in the figure below.

What happened to these three species on the island and why? How did these species interact with each other and with the rest of the ecosystem? Justify your answer.



Population density of the three beetle species A (red), B (blue), and C (green).

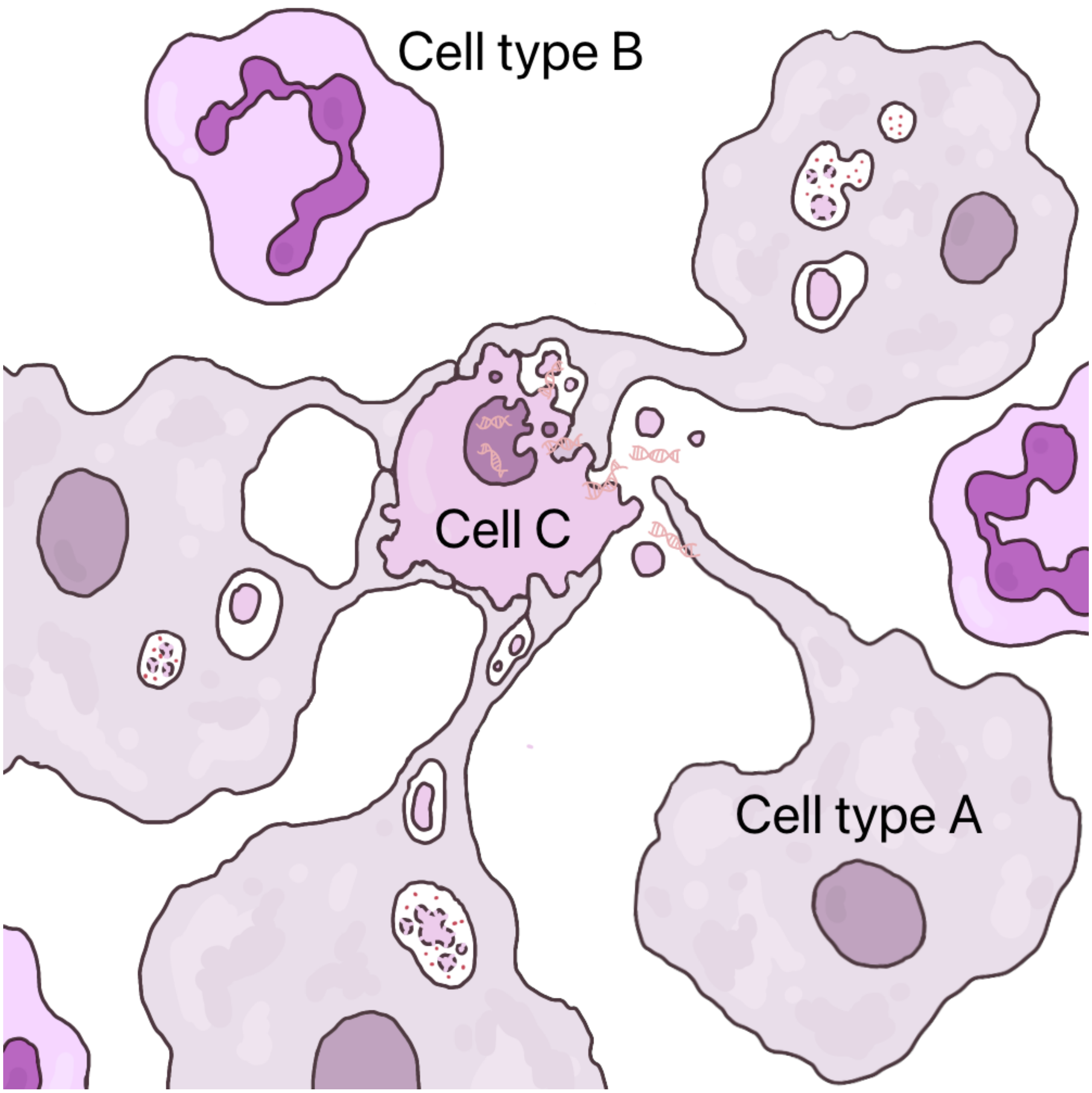
Hint:

Consider a situation when some of those species do interact with others.

Solution:

Answer: "C" demonstrates typical behavior of an invasive species. It is unlikely that "C" interacted with "A" and "B". "A" and "B" are prey and predator, respectively. Most likely, that "A" is "B's" natural prey, and "B" is a very efficient and highly specialized predator that quickly destroyed "A", thereby undermining its own food supply.

10 points:



This depicts **a biological process involving only human cells**.

Referencing the diagram above:

(a) Identify and explain the process depicted, citing evidence from the diagram.

(b) Explain the role played by the cells of cell type A in this process, and provide two reasons why it is important.

(c) Explain how this process would be different if Cell C were a pathogen.

Hint:

Read about "phagocytosis" and "chemotaxis". Can you figure out what the type A cells are? Also, why don't the type B cells seem to be doing much, and why is that a good thing?

Solution:

The key to solving this problem is to realize that cell type A refers to *macrophages*. These are large immune cells that specialize in engulfing and digesting pathogens (such as bacteria), damaged or dead cells, and any debris, as well as regulating inflammation. Since macrophages are so widespread and found basically everywhere in the body, they are your first line of defense against damage or infection, as well as under healthy conditions with dying or dysfunctional cells, as in this case.

Macrophages have a number of properties that are depicted in the diagram. Aside from their large size, macrophages can produce *pseudopods*, which are long, appendage-like extrusions of the cell. Macrophages can also perform *phagocytosis*, a process by which the cell traps and engulfs some material in its lipid membrane, which then pinches off into an internal compartment called a *phagosome*. Then, the phagosome merges with another compartment called a *lysosome*, which contains enzymes that degrade and digest the material. Macrophages also have the ability to move towards certain stimuli, a process called *chemotaxis*. Specifically, macrophages are attracted by many molecules typically found only inside cells, like DNA, because they are signs of cellular damage and possibly infection. All of these properties of macrophages are shown in the diagram.

a) The process depicted in the diagram is the gathering of macrophages, labeled as cell type A, around a damaged or dead human cell, in order to "clean up" the debris. The processes of chemotaxis and phagocytosis are also depicted.

b) The macrophages are surrounding the destroyed Cell C and engulfing fragments of it via phagocytosis. There are a number of reasons why this is important, including:

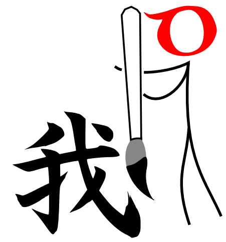
    – By digesting dead cells, macrophages recycle molecules, resources, and debris left behind.

    – By surrounding and "cloaking" the cell, macrophages can prevent the debris from attracting *neutrophils* (cell type B), another type of immune cell that causes inflammation and can release cell-killing toxins, even when there are no pathogens, which causes additional damage to healthy cells.

    – Macrophages (specifically *resident-tissue* macrophages, your first responders to tissue damage), can promote tissue repair by secreting anti-inflammatory molecules and removing dead cells to make space for new healthy tissue, among other mechanisms.

c) If Cell C were a pathogen, the macrophages would not aim to conceal debris from neutrophils and would encourage inflammation. The neutrophils would also be performing phagocytosis, as well as undergoing *degranulation* (releasing toxic compounds to destroy the pathogen), or engaging *neutrophil extracellular traps (NETs)* (releasing "nets" of their own DNA to trap and kill pathogens).

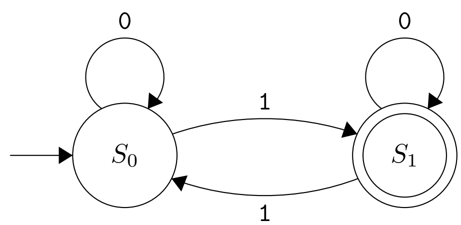
Linguistics & Applied Sciences



Have you ever wanted to try your hand at writing your own POM problem? This month, we are introducing the **Linguistics / Applied Science POM Participant Challenge**! Write and submit your own POM problem and solution by **October 10th** and it might just be published alongside our October problems.

- For inspiration, you can take a look at problems from previous months! We're looking for well written problems that are fun to solve.
- Submit your problem by emailing a PDF of the problem and solution to pom@sigmacamp.org.
- If you have any questions, you can contact pom@sigmacamp.org.

Automata theory is the study of *automata*, which are computational models used in many different areas of computer science, including the design of programming languages, artificial intelligence, and computational linguistics. This month, we will explore basic examples of automata, and how they can be used for linguistics.



An *automaton* consists of *states* and *transitions* between the states. In the above automaton, the state S_0 is the *initial state*, indicated by the arrow going into it on the left. The state S_1 is a *final state*, indicated by it being double-circled. The transitions are arrows between the states, and are labelled by `0` and `1` in this example.

A *string* is a finite sequence of characters. For the above example where the characters are `0` and `1`, an example of a string is "`011110101`". An automaton *reads* a string from left to right and follows the transitions, starting from the initial state. If the automaton finishes reading the string and ends on a final (double-circled) state, we say that the automaton *accepts* that string.

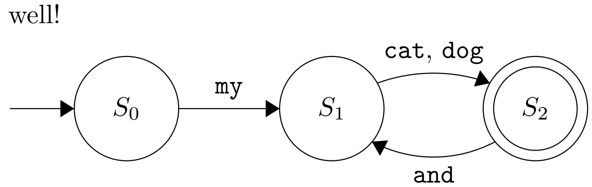
For example, the string "`01`" is accepted by the above automaton, as the automaton starts at S_0, reads `0` and loops back to S_0, then reads `1` and goes to S_1, which is a final state.

Some other strings that are accepted by this automaton (try it out for yourself!) are: `111`, `1000`, `1101101`.

Some strings that are *not* accepted are: `11`, `110101`, `11010101001`, and the empty string.

After a bit of thought, we can see that the above automaton accepts a binary string if and only if the number of `1`s in the string is *odd*.

Now let's see how we can use automata in linguistics. Instead of having our alphabet be numbers or letters, we can make them words as well!



In the above automaton, the transition with the label "`cat, dog`" allows both "`cat`" and "`dog`".

Some strings accepted by this automaton are (check it for yourself!):

- my dog
- my cat and dog
- my dog and dog and cat and dog
- my cat and cat and cat and cat and dog and cat and dog and cat and dog

Some strings that are *not* accepted by this automaton are (check it for yourself!):

- cat
- dog my
- my and cat dog
- my dog dog
- the empty string

A concise way of describing all of the strings that are accepted by this automaton is:

my <dog/cat> (and <dog/cat> ... and <dog/cat>)

where the contents inside of the parentheses ( ) are optional.

5 points:

Design and **draw** an automaton that accepts strings of the form

my <dog/cat> <eats/sleeps> (and <eats/sleeps> ... and <eats/sleeps>)

where the contents inside of the parentheses ( ) are optional.

For example, some strings your automaton should accept are:

- my dog sleeps
- my cat eats and sleeps
- my dog eats and eats and eats and eats and eats
- my cat sleeps and eats and eats and sleeps
- my cat eats and sleeps and eats and sleeps and eats and sleeps and eats and sleeps

Some strings that are should *not* be accepted are:

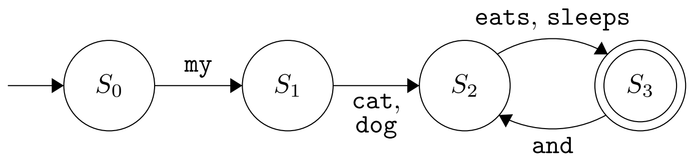
- cat sleeps
- my dog eats my cat
- my sleeps cat my dog
- dog cat sleeps my cat dog eats cat eats dog eats sleeps my
- the empty string

Make sure your automaton includes an initial state and a final state, and test your automaton on the above examples and come up with some other ones to ensure that it is correct.

Hint:

No hint this month.

Solution:



10 points:

Design and **draw** an automaton that accepts strings of the form

```
the (<shy/cute>) <cat/dog> (<likes/hates> the (<shy/cute>) <cat/dog> (who <likes/hates>
    the (<shy/cute>) <cat/dog> ...  who <likes/hates> the (<shy/cute>) <cat/dog>))
```

where the contents inside of the parentheses `( )` are optional.

For example, some strings your automaton should accept are:

- `the cat`
- `the shy dog`
- `the shy cat likes the dog`
- `the cat likes the shy dog`
- `the dog likes the cute dog who hates the shy cat`
- `the shy cat hates the dog who likes the cat who likes the shy cat`
- `the cat likes the dog who likes the cute cat who hates the shy dog who likes the cute dog`

Some strings that are should *not* be accepted are:

- `the`
- `shy the dog`
- `the cute shy dog`
- `the dog hates the`
- `the dog likes the cat who`
- `the dog the shy cat`
- the empty string

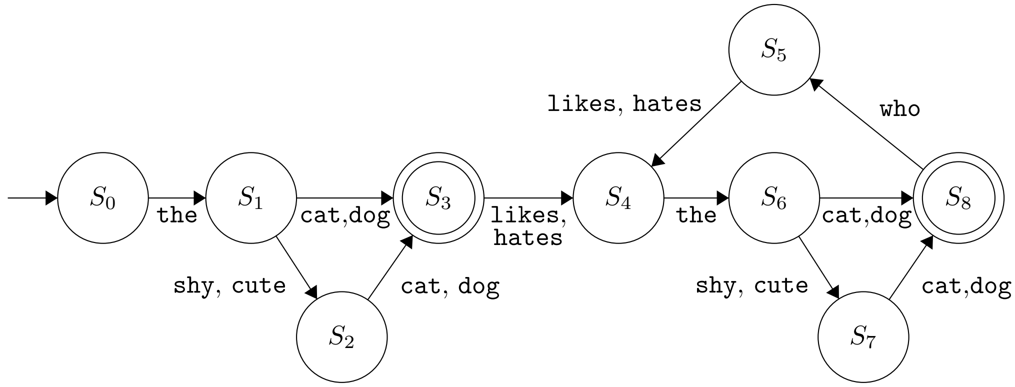
Make sure your automaton includes an initial state and a final state, and test your automaton on the above examples and come up with some other ones to ensure that it is correct.

Your automaton may have more than one final state.

Hint:

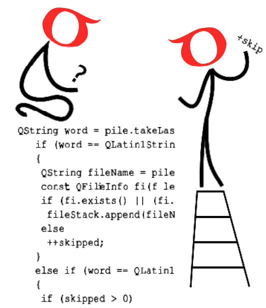
No hint this month.

Solution:



S_0
the
S_1
cat,dog
S_3
shy, cute
S_2
cat, dog
likes,
hates
S_4
the
S_6
cat,dog
S_8
shy, cute
S_7
cat,dog
S_5
likes, hates
who

Computer Science



- Your program should be written in Java or Python-3
- No GUI should be used in your program (e.g. `easygui` in Python).
- All the input and output should be done through files named as specified in the problem statement.
- Java programs should be submitted in a file with extension `.java`; Python-3 programs should be submitted in a file with extension `.py`.
  No `.txt`, `.dat`, `.pdf`, `.doc`, `.docx`, etc. Programs submitted in the incorrect format will not receive any points!

We know that sometimes it can be difficult to know where to start with CS problems, so we are offering a **September Problem Solving Session!** Join us for a fun video call where we will solve a 5 point problem from a past month together, and go over some coding tips. This session will be offered on **Saturday, September 30th at 2 PM EST**, and you can join with this link: https://gatech.zoom.us/j/93510382107. Hope to see you there!

Have you ever wanted to try your hand at writing your own POM problem? This month, we are introducing the **CS POM Participant Challenge**! Write and submit your own POM problem and solution by **October 10th** and it might just be published alongside our October problems.

- For inspiration, you can take a look at problems from previous months! We're looking for well written problems that are fun to solve.
- Submit your problem by emailing a PDF of the problem statement and the Python or Java file of your solution to pom@sigmacamp.org.
- If you have any questions, you can contact pom@sigmacamp.org.
- While no points will be awarded for the Participant Challenge, depending on interest participants who have their problems published may receive a small prize.

Anna, who is working as a research assistant over the summer, is studying bacteria colony growth. For her research project, she grows bacteria colonies in a Petri dish and analyzes their characteristics. Anna wants to write a computer program to analyze the bacteria colonies. She represents a petri dish using an m-by-n grid with either a "`#`" (representing a bacterium) or "`.`" (representing empty space). Below is an example of a 10-by-4 petri dish:

```
..#.......
#.##....#.
.....#....
#.##..####
```

Anna is tasked to study the number of isolated bacteria in a petri dish, as well as the connectedness of bacteria colonies. A *bacteria colony* is a group of bacteria that are all connected to each other by a path of adjacent (horizontally, vertically, or *diagonally*) bacteria. In the below petri dish, all of the bacteria colonies are highlighted with a different color:

```
..#.......
#.##....#.
.....#....
#.##..####
```

An *isolated bacterium* is one that is not adjacent to another bacterium (i.e., it is the only bacterium in its colony). In the below petri dish, the isolated bacteria are highlighted:

```
..#.......
#.##....#.
.....#....
#.##..####
```

This month, the 5 and 10 point problems will use the same input format. Your program should receive the input file `input.txt`, which will consist of $n + 1$ lines. The first line will contain the width (m) and the length (n) of the petri dish separated by a space. The next n lines will contain m characters each, with each character either being "`#`" (representing a bacterium) or "`.`" (representing empty space).

5 points:

Write a program that receives the input as described above and computes the number of isolated bacteria within the petri dish. Your program should produce the output file `output.txt`, containing one line with the integer number of isolated bacterium.

Hint:

No hint this month.

Solution:

The 5pt solution is available on the SigmaCamp GitHub repository here:

https://github.com/SigmaCode/POM-QQ/tree/main/CS/2023-2024/September

10 points:

Write a program that receives the input as described above and computes the optimal location where a single bacterium should be added to the dish to maximize the size of the resulting largest bacteria colony.

If there is more than one position (x, y) where the bacterium can be placed, your program should compute the "least" such position: if two different positions (x_1, y_1) and (x_2, y_2) are both optimal and $x_1 < x_2$, compute (x_1, y_1). If $x_1 = x_2$ and $y_1 < y_2$, compute (x_1, y_1). Otherwise, compute (x_2, y_2). The top-left corner of the petri dish is $(0, 0)$.

Your program should produce the output file `output.txt`, containing one line with *two space-separated integers* x and y, which are the horizontal and vertical positions where adding a bacterium results in the largest possible bacteria colony.

Hint:

Read about "breadth-first search", and consider how it may be used to find the size of the largest bacteria colony.

Solution:

The 10pt solution is available on the SigmaCamp GitHub repository here:

https://github.com/SigmaCode/POM-QQ/tree/main/CS/2023-2024/September

See the examples on the next page.

Examples:

Sample 1:

`input.txt`:

```
10 4
..#.......
#.##....#.
.....#....
#.##..####
```

(5pt)
`output.txt`:

```
3
```

There are three isolated bacterium in the above dish, at $(0, 1)$, $(0, 3)$, and $(8, 1)$.

(10pt)
`output.txt`:

```
4 2
```

Placing a bacterium at $(4, 2)$ results in the largest bacteria colony.

Sample 2:

`input.txt`:

```
5 6
#.#.#
.....
.#.#.
.####
#....
#.#.#
```

(5pt)
`output.txt`:

```
5
```

There are five isolated bacterium in the above dish: $(0, 0)$, $(2, 0)$, $(4, 0)$, $(2, 5)$, and $(4, 5)$.

(10pt)
`output.txt`:

```
1 1
```

The other possible solutions are $(3, 1)$ and $(3, 4)$, but the x value of $(1, 1)$ is the smallest, making it the correct output.

When we grade your submission, we test it on test cases just like these examples, so these examples are great ways to validate your responses to make sure your code is working the way it should.