



# SigmaCamp Qualification Quiz 2022

This Qualification Quiz is only one part of your application to SigmaCamp. For full instructions please check <http://sigmacamp.org/2022/apply>.

The Qualification Quiz contains two problems from each of the six main disciplines at Sigma – math, physics, chemistry, biology, computer science, and linguistics. The second problem in each category was designed to be more challenging than the first problem. **You are not expected to solve all the problems.** We will evaluate your quiz submission based on your approach to the problems and the quality of reasoning. You can use the Internet, books and even help from someone, but **state precisely what sources you have used to solve each problem.** Note that you **cannot** repost the problems to any Internet or other public forums and solicit help that way. Also, please do not collaborate with other applicants.

1. Your solutions should be submitted either as PDF, plain text (.txt) or Word document files (.doc or .docx), with the exception of Computer Science, where it should be .java or .py files (see Computer Science section below for more details).
2. Your solutions can be handwritten or typed. Handwritten solutions must be scanned as PDF (not as JPG or PNG) files.
3. Files containing solutions should be named as follows: *Subject.ProblemNumber.Your name.extension*, where Subject is Math, Physics, Biology, Chemistry, CS or Linguistics. ProblemNumber can be 1 or 2, corresponding to the problem number in the corresponding subject. For example, here are valid file names: Chemistry.1.John Smith.pdf, Biology.2.Jane.Miller.doc, CS.1.SamJohnson.java.
4. Each solution should be submitted in a separate file. Please don't combine the solutions for problems 1 and 2 of the same subject into one file, and also don't combine the solutions across subjects.
5. If you were accepted to Sigma through the Problem of the Month contest (PoM), you do not need to submit the QQ. **If you participated in PoM, but were not admitted among the top scorers, your effort will not be forgotten: 30% of your cumulative PoM score will be added to your QQ score.**

**The application deadline is April 15, 2022** - all your materials (Qualification Quiz, essay, letters of recommendation) must be submitted by that date. We will notify applicants regarding an acceptance decision no later than **May 5th**.

**Good luck with your application!**

## Math

### Problem 1. (5 pts)

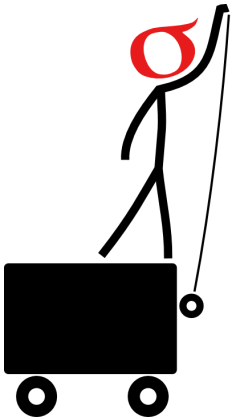
Find all positive integers  $N$ , no greater than 2022, such that  $N^2 + 1$  campers can be equally divided into  $N + 1$  teams. For full score, mere listing will not suffice, and a proof of completeness of the answer will be needed.



### Problem 2. (10 pts)

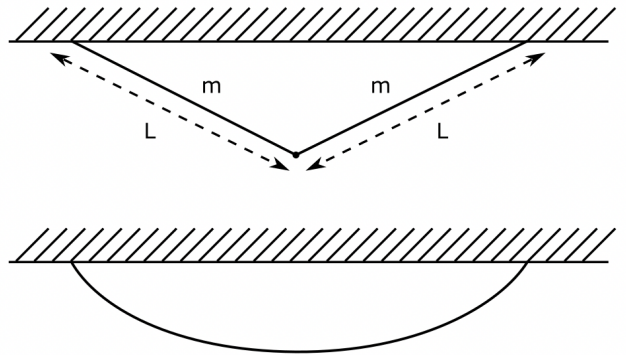
An ancient ruler who was believed to be very rich (but was nearly broke) told his subjects that anyone can come and get 10% of the ruler's wealth in exchange for 10 coins. After very many exchanges, he managed to increase his wealth twofold. What was his original wealth?

## Physics



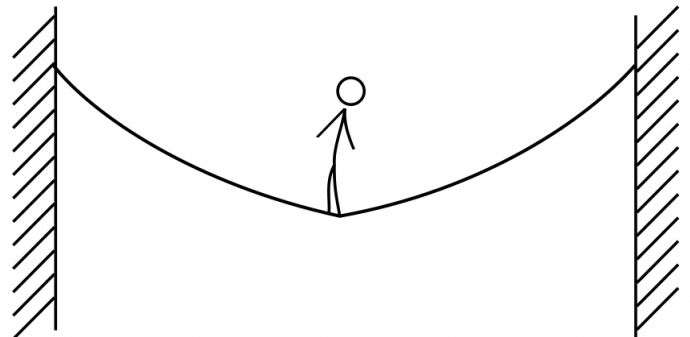
### Problem 1. (5 pts)

Consider two objects: 1) A pair of rods each length  $L$  and mass  $m$  connected at a pivot. 2) A rope of length  $2L$  and mass  $2m$ . The ends of both objects are fastened on the ceiling and the objects hang freely. Which system has a higher center of mass, and why?



### Problem 2. (10 pts)

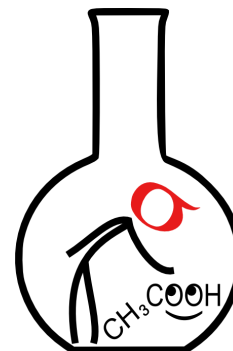
In the high rope course, when a person weighing  $M=50\text{kg}$  is in the middle of the rope, the angle the rope makes with itself at the midpoint is  $120^\circ$  (see Figure). Will the rope hold if it breaks when tension (at any point of the rope) is greater than 100 kg, and the rope itself weighs  $m=25\text{kg}$ ?



# Chemistry

## Problem 1. (5 pts)

To broaden international outreach, Sigma Camp was held in Yellowknife, Canada, in February of 2022. The day of arrival was unusually warm with snow melting around the cabins. Early next morning, the administrative team got up to find that everything outside was covered with ice. Unfortunately, the bins with deicer were empty. One representative from each of the cabins went to a local hardware store to find a suitable substitute. Due to a combination of communication difficulties, language barrier, and stubbornness, they all purchased different substances as shown in the table below. The temperature that day was not forecasted to rise above  $-25^{\circ}\text{C}$ . Each responsible person came back and applied their chosen substance generously around their cabin. The ice melted readily around one cabin. Another cabin was able to melt the ice but only after applying a large amount of their substance. Three other cabins were not successful that day in melting the ice. Please identify the cabins that were able to melt the ice and explain your reasoning.



Cabin	Substance
Marmot	Sodium chloride
Ekati	Potassium chloride
Ptarmigan	Calcium chloride
Diavik	Urea
Mounties	Ammonium nitrate

## Problem 2. (10 pts)

The central question during planning for Sigma Camp 2022 was which car to buy as an official Sigma Camp vehicle. During Sigma Camp 2021, campers overwhelmingly voted in favor of getting a car with the smallest carbon footprint. Currently, five vehicles are under consideration that use the following fuel types:

- 1) electric; 0.35 kWh/mile
- 2) hydrogen fuel cell; 1.34 kg/100 miles
- 3) natural gas; 20 miles/100 SCF (standard cubic foot)
- 4) gasoline; 28 mpg, fuel density 0.76 kg/L
- 5) diesel; 35 mpg, fuel density 0.85kg/L

1 SCF is one cubic foot of gas at standard conditions. In the Sigma Camp world headquarters area, the electricity is generated from natural gas (methane), and it requires 7.43 SCF per one kWh). Assume 30% loss of electricity during transmission and charging. Currently, 95% of the world's hydrogen is produced by natural gas reforming, an endothermic process that converts a mixture of methane and water vapor to carbon dioxide and hydrogen. The energy for that process is obtained from fossil fuel combustion, and it leads to production of additional 7.6 metric tons of carbon dioxide per million SCFs of hydrogen.

Calculate the carbon footprint, measured in annual emission of carbon dioxide, for each car assuming that they would drive 12,000 miles per year. Which car should Sigma Camp get?



## Biology

### Problem 1. (5 pts)

You visit your Grandma and, after chatting for a bit, conclude that she is starting to show some signs of age-based cognitive decline. You remember hearing, during a recent Sigma lecture, that while people tend to think of dementia in terms of "memory loss," structural MRI suggests that the first signs of brain atrophy are not of the *hippocampus* (associated with memory) but rather of the *ventral striatum* (associated with learning). This suggests the hypothesis that dementia might not be a problem of information *retrieval*, but rather one of information *encoding*.

Expanding specificity even further, you'd like to know, if your Grandma doesn't remember something you just said a few minutes later, is this because:

- - she has difficulty hearing?
- - she can hear but she can't make sense of what she heard?
- - she can make sense but it doesn't "stick" in short-term memory (meaning, it's not encoded)?
- - it sticks in short-term memory, but she can't then access it later (retrieval problem)?

Since you have no access to MRI and biosensors equipment, you decide to improvise by developing a clever diagnostic, requiring no equipment and requiring no more than 10 steps, by which to identify the precise point in the above four stages of cognitive processing that is most responsible for her impairment. Please remember that, to dissociate each of these steps, you'll need to carefully control for other variables.

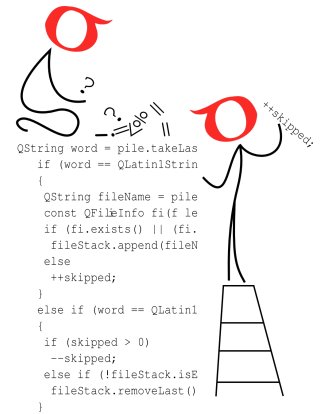
### Problem 2. (10 pts)

According to Carbon Dioxide Information Analysis Center (<https://cdiac.ess-dive.lbl.gov/pns/faq.html>), people exhale carbon dioxide at the rate of approximately 1 kg per day, although it depends on the person's diet and activity level. Estimate if that figure is consistent with what we know about human metabolism, diet, the principle of mass conservation and the law of conservation of energy.

# Computer Science

- Your program should be written in Java or Python-3
- No GUI should be used in your program: eg., easy gui in Python
- All the input and output should be via files with specified in the problem statement
- Java programs should be submitted in a file with extension .java; Python-3 programs should be submitted in a file with extension .py.  
**No .txt, .dat, .pdf, .doc, .docx, etc. Programs submitted in incorrect format will not receive any points!**

**Please briefly describe your algorithm in the comments at the top of your program.**



## Problem 1. (5 pts)

Table tennis has the following rules:

- A Game is played to 11 points.
- A Game must be won by two points. If the two-point difference is not attained, the Game can continue over the 11-point maximum until a two-point difference is reached.
- A Match is the best three of five Games: the winner of three Games is the overall winner of the Match (which stops at that point).

Your program will need to validate the play-by-play progression of the score of a table tennis match. In the input file **input.txt** it will be given a series of progressive scores, presented as a series of space-separated pairs of numbers, line by line, and where the first number corresponds to the first player and the second number. Here is an example of a snippet from the input file:

```
...
7 9
7 10
8 10
8 11
1 0
1 1
...
```

The input is supposed to contain the full score progression of the whole match. Your program will need to validate it and provide the following output to **output.txt** file:

- if the input contains the full score progression of the whole match, write:

CORRECT

- if you find a violation in the progression, write:

ERROR LINE n

where n is the line number of incorrect score progression detected

- if you find that the match record is not complete (for example, if it stops after only two games being won), write:

INCOMPLETE

## Problem 2. (10 pts)

*Please refer to the 5-point problem for the rules of table tennis.*

N Sigma campers called P1...Pn signed up for a table tennis tournament. The tournament was supposed to be played as a round-robin tournament meaning that each player would play each other player once. However, 2 rounds before the completion of the tournament a strong thunderstorm arrived, and the tournament had to be stopped and was not finished. Considering that the result of the match is always pre-determined by the relative strength of the players, and the stronger player always wins, your program should help the tournament organizers to calculate all possible final standings for the tournament according to relative player strength - from strongest to weakest.

Your program will receive the input file named **input.txt**, which contains the records of all of the completed matches in the tournament in the following space-separated format:

PlayerA PlayerB #ofGamesWonByPlayerA #ofGamesWonByPlayerB

one match per line. For example:

P1 P9 1 3

P2 P4 3 0

Your program should write to the output file **output.txt** the following:



- if it could calculate the standings, it should output all the possible standings in lexicographic order, one standing per line, where each standing is the list of players, from strongest to weakest, in a space-separated format, for example:

```
P1 P2 P3 ... P12
```

```
P2 P1 P3 ... P12
```

- if the tournament results are inconsistent with the conditions stated above, for example, if it is detected that a stronger player always wins rule is not obeyed, write:

```
INCONSISTENT
```

- if incorrect scores are detected, write:

```
INCORRECT SCORE LINE n
```

where n is the line number of the detected incorrect score

You can assume that there are no more than 20 players participating in the tournament.

# Linguistics

## Problem 1. (5 pts)

Below are five sentences written in a constructed language. Match them to their translations.

1. u iril nivi ruitir
2. u mottyl ol iril nivi ril lhitir
3. u mottyl piniilk hysriv hitol
4. u iril dutilk hysriv y pottyl
5. u 'itir ylik hysriv oriul

- a. The man took an apple from a friend
- b. The friend was eating a green apple
- c. The man sits beneath a tree
- d. The tree gave an apple to men
- e. The friend of a man sits against a tree

Render the following sentence into this constructed language:

The man gives an apple from a green tree

Use what you have learned about letter mutations and the given information to render the following sentence into this constructed language:

The women walk to a road

Hint:

imis - woman

pol - road

dilrsi- walk

lil - to (a location)

-s - the 3rd person plural verb ending

## Problem 2. (10 pts)

Three SigmaCampers taking an introductory Computer Science semilab decide to make a program that sorts a list of sentences into a whole conversation, aptly named “ChatCat”. They test their program by inputting the following shuffled transcript of one of their conversations from the night before:

### SENTENCE BANK

The townspeople won.  
I was part of the Mafia.  
I would like to go with you to Mafia.  
Mafia is way more fun.  
Sorry I didn't make it to ballroom dance lessons, I went to play Mafia.  
That's alright.  
Who won the game?  
I signed up for the escape room tomorrow.  
You're wrong!  
Do you both want to come play with me?  
I want to play again tomorrow.  
You should try ballroom before the dance on the last night.

First, they program ChatCat to order the sentences by increasing character count (and breaking ties, if any, in alphabetical order) to see if that puts the conversation in a logical order. The result of their program is shown below:

### **Example Output**

A: You're wrong!  
B: That's alright.  
C: Who won the game?  
A: The townspeople won.  
B: Mafia is way more fun.  
C: I was part of the Mafia.  
A: I want to play again tomorrow.  
B: I would like to go with you to Mafia.  
C: Do you both want to come play with me?  
A: I signed up for the escape room tomorrow.  
B: You should try ballroom before the dance on the last night.  
C: Sorry I didn't make it to ballroom dance lessons, I went to play Mafia.

After deciding that the first version of ChatCat was inadequate, they decided to have a contest to see who could write the best algorithm to assemble a conversation. Unfortunately, their programs can still only interpret sentences as strings of characters or words without any meaning or context.

**Part I: Describe how each of their ChatCats below orders sentences into conversations.**

**ChatCat #1**

A: Do you both want to come play with me?

B: I signed up for the escape room tomorrow.

C: I want to play again tomorrow.

A: I was part of the Mafia.

B: I would like to go with you to Mafia.

C: Mafia is way more fun.

A: Sorry I didn't make it to ballroom dance lessons, I went to play Mafia.

B: That's alright.

C: The townspeople won.

A: Who won the game?

B: You should try ballroom before the dance on the last night.

C: You're wrong!

**ChatCat #2**

A: You're wrong!

B: That's alright.

C: The townspeople won.

A: Who won the game?

B: Mafia is way more fun.

C: I was part of the Mafia.

A: I want to play again tomorrow.

B: I signed up for the escape room tomorrow.

C: I would like to go with you to Mafia.

A: Do you both want to come play with me?

B: You should try ballroom before the dance on the last night.

C: Sorry I didn't make it to ballroom dance lessons, I went to play Mafia.

**ChatCat #3**

A: I was part of the Mafia.

B: Mafia is way more fun.

C: Do you both want to come play with me?

A: Who won the game?

B: I would like to go with you to Mafia.

C: Sorry I didn't make it to ballroom dance lessons, I went to play Mafia.

A: I want to play again tomorrow.

B: I signed up for the escape room tomorrow.

C: You should try ballroom before the dance on the last night.

A: You're wrong!

B: The townspeople won.

C: That's alright.

**Part B:** The three campers then decide to add three new sentences to the sentence bank and run their ChatCats again. How will each ChatCat add the sentences below into its existing conversation, and explain why:

- (1) Ballroom is my favorite evening club.
- (2) I wish I had gone to Mafia yesterday too.
- (3) Wow, that's cool!

**Part C:** The three friends share their revolutionary invention with one of their teammates who is taking a linguistics semilab. Can you come up with an algorithm that better orders sentences into conversations, only considering sentences as strings of characters or words without any meaning or context? Show the output of your algorithm given the provided sentence bank. If your ChatCat could use an AI that could understand semantic context and similarity, how would you try to improve your algorithm?

**The application deadline is April 15, 2022.** We will notify applicants regarding acceptance decisions no later than **May 5th**.

Good luck with your application!