SigmaCamp Qualification Quiz 2023

This Qualification Quiz is only one part of your application to SigmaCamp. For full instructions please check http://sigmacamp.org/2023/apply.
The Qualification Quiz contains two problems from each of the six main disciplines at Sigma – math, physics, chemistry, biology, computer science, and linguistics. The second problem in each category was designed to be more challenging than the first problem. **You are not expected to solve all the problems**. We will evaluate your quiz submission based on your approach to the problems and the quality of reasoning. You can use the Internet, books and even help from someone, but **state precisely what sources you have used to solve each problem**. Note that you **cannot** repost the problems to any Internet or other public forums and solicit help that way.

Additionally, you cannot collaborate with other applicants.

1. Your solutions should be submitted either as PDF or plain text (.txt) files, with the exception of Computer Science, where it should be .java or .py files (see Computer Science section below for more details).

2. Your solutions can be handwritten or typed. Handwritten solutions must be scanned as PDF (not as JPG or PNG) files.

3. Files containing solutions should be named as follows: *Subject.ProblemNumber.Your name.extension*, where Subject is Math, Physics, Biology, Chemistry, CS or Linguistics. ProblemNumber can be 1 or 2, corresponding to the problem number in the corresponding subject. For example, here are valid file names: Chemistry.1.John Smith.pdf, Biology.2.Jane.Miller.doc, CS.1.SamJohnson.java.

4. Each solution should be submitted in a separate file. Please don't combine the solutions for problems 1 and 2 of the same subject into one file, and also don't combine the solutions across subjects.

5. If you were accepted to Sigma through the Problem of the Month contest (PoM), you do not need to submit the QQ. **If you participated in PoM, but were not admitted among the top scorers, your effort will not be forgotten: 30% of your cumulative PoM score will be added to your QQ score.**

The application deadline is April 15, 2023 - all your materials (Qualification Quiz, essay, letters of recommendation) must be submitted by that date. We will notify applicants regarding an acceptance decision no later than **May 5th**.

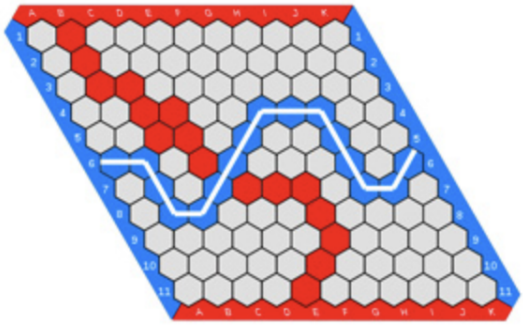
Good luck with your application!

Math

Problem 1. (5 pts)

The game of Hex is a two-player game in which each player places hexagonal tiles on the board and tries to complete a path from one edge of the board to the opposite edge, while preventing their opponent from doing the same on their own side. In this game, there can be no ties. This is simple to conceptualize, as having a path between opposite sides of a single color necessarily means that there can't be a path from the other pair of sides of the other color. Proving this rigorously takes some more work, and if you're curious you can read up here: https://web.mit.edu/sp.268/www/hex-notes.pdf. (Note: reading this document is not necessary to solve the problem below).

Imagine you have a 5x5 Hex board and no opponent. Let us define an efficient path as a path which uses the fewest number of tiles possible to get from its starting point to its ending point. How many efficient paths are there between two opposite edges of the board? Note that on a 5x5 grid, all efficient paths from one edge to the opposite one will use exactly 5 tiles.

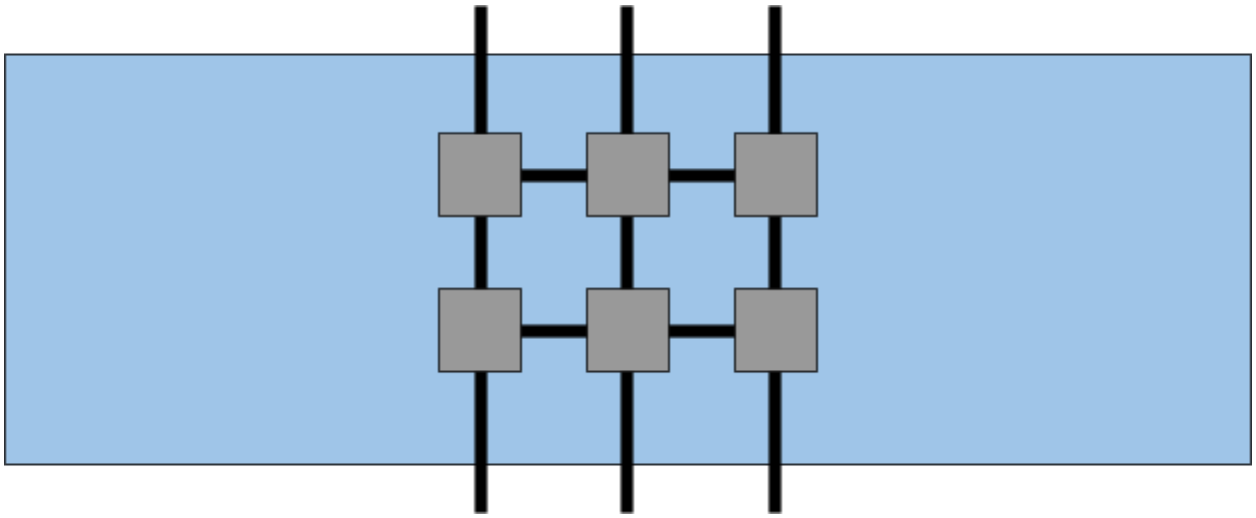


A game of hex won by blue on an 11x11 board. Note that blue's path is not efficient!
Credit: Jean-Luc W

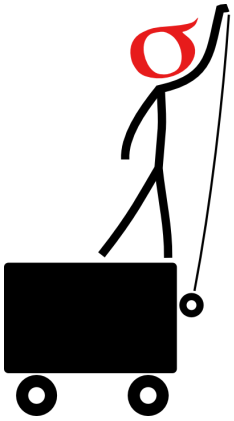
Problem 2. (10 pts)

You wish to cross a dangerous river! Unfortunately, the only way across is through a network of 13 bridges between the two riverbanks and 6 platforms (pictured below) created by a malevolent wizard. Each bridge is fitted with a gate. Each day, the wizard decides which bridges will be traversable by randomly deciding whether to open or close each bridge's gate. Each day, each gate will be open with probability 50% and closed with probability 50%. These probabilities are all independent. What is the probability that you will be able to cross the river on any given day via some path in the network of bridges?

Each black line corresponds to a gated bridge, the gray squares are junctions, and the blue rectangle is the river you must cross. You are free to walk around on the riverbanks and junctions as you please.

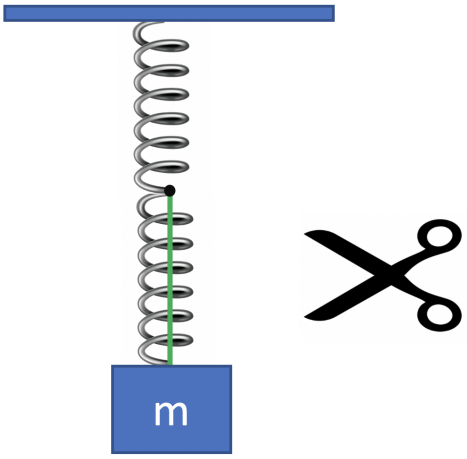


Physics



Problem 1. (5 pts)

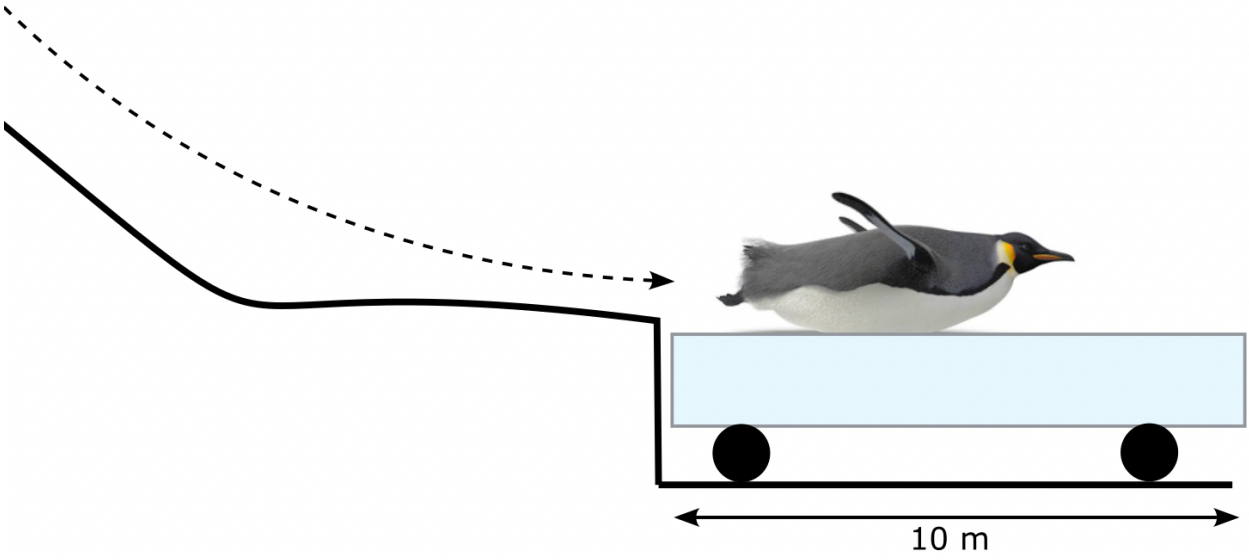
A weight of mass m hangs on two identical (massless) springs (spring constant k) attached together at a point (marked with a black dot in the image below). While the system is at rest, we tie a string between the ends of the bottom spring. At time $t = 0$ we then pull the weight down by distance A, and release it. The system will begin to oscillate. After 2.5 periods of oscillation, when the weight is at the highest point, we cut the string, and allow the system to continue its oscillation. Describe the motion of the system, i.e., show how the position of the weight will move as a function of time, starting at $t = 0$ when the weight is released.



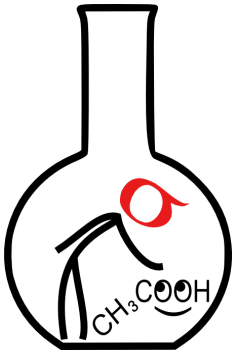
Hint: A simple mass-on-spring harmonic oscillator, with a weight of mass m on a spring with spring constant k, will oscillate sinusoidally with period $T = 2\pi\sqrt{m/k}$. Ref: Wikipedia

Problem 2. (10 pts)

Consider an amusement park for penguins, with a ride that begins with a slide and ends with a 10 meter long ice block on frictionless wheels. A penguin slides down the slope and lands onto the ice block. At the start of the ice block, the penguin is moving with horizontal velocity v, and the ice block is stationary. The dynamic coefficient of friction between the penguin's belly and the ice is $\mu = 0.1$. What is the final velocity v_f of the penguin and ice block after the penguin stopped skidding? What is the maximum safe velocity for this ride (the maximum v for which the penguin stops before falling off the front of the ice block)? The mass of the penguin is $m = 5$ kg, the mass of the ice block is $M = 20$ kg.



Chemistry



Problem 1. (5 pts)

4 g of an aluminum-magnesium alloy were dissolved in 300 mL of 3 M HCl. The volume of the gas formed in this reaction was 4.27 L. Calculate the composition of the alloy.

Problem 2. (10 pts)

Ideal catalysts increase the rate of a chemical reaction, while remaining unchanged. In reality, catalysts gradually deteriorate, so the actual number of catalytic cycles (i.e. the number of moles of converted material per one mole of catalyst) is finite. Chemists call this number the "catalyst turnover number": it is defined as the number of catalytic cycles performed by the catalyst before it deteriorates.

2.187 g of water soluble catalyst **A** (molecular weight 243 Da) were added to 500 mL of 30% hydrogen peroxide. The gas that formed in this reaction was collected. After the reaction ceased, the volume of the gas was 11.1 L.

Calculate the catalytic turnover number of **A.**

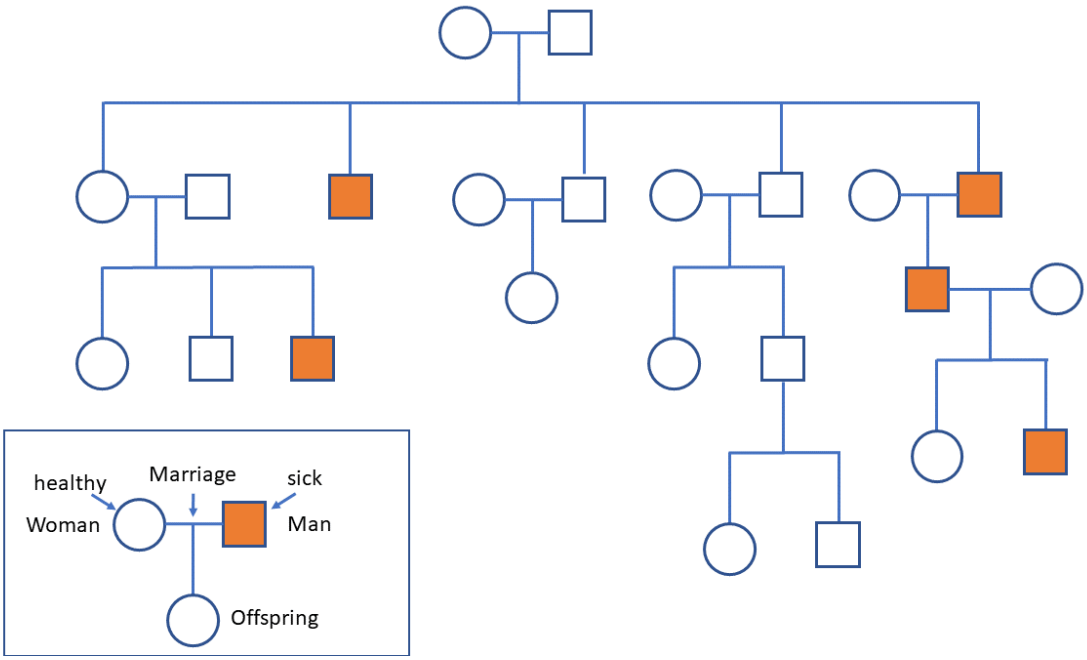
Biology

Problem 1. (5 pts)

A healthy man and a healthy woman had 5 children, some of them were suffering from some strange disease, whereas others were totally healthy. This disease reappeared in next generations, as shown on the scheme below. Assuming that this disease is caused by a single mutation in a single gene, answer the following questions:

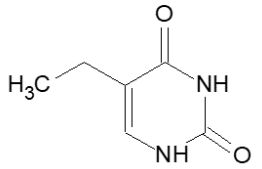
1. In which chromosome the mutated gene is located?
2. Is that trait recessive or dominant?
3. Who, a mother or a father, brought this disease to the family?

Explain your answers.

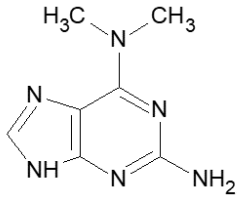


Problem 2. (10 pts)

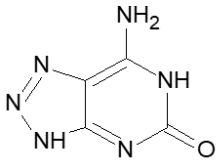
A spaceship "Tantra" arrived at the Earth-like planet orbiting around a yellow dwarf near the Orion nebula. A brief analysis revealed the presence of life on that planet. Scientists obtained samples of some tree-like object, and they extracted some material from it that, by its chemical and physical properties, was pretty similar to our DNA. An acid hydrolysis of this sample yielded a mixture of the following heterocycles (their abundance in the sample is shown in parentheses):



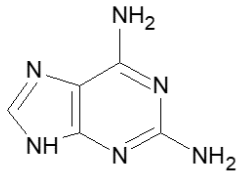
1 (9.89%)



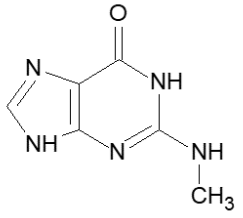
2 (3.50%)



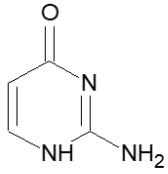
3 (14.19%)



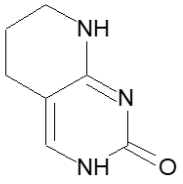
4 (9.77%)



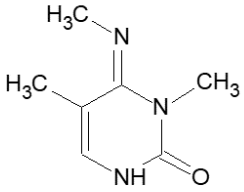
5 (22.50%)



6 (14.10%)



7 (22.71%)

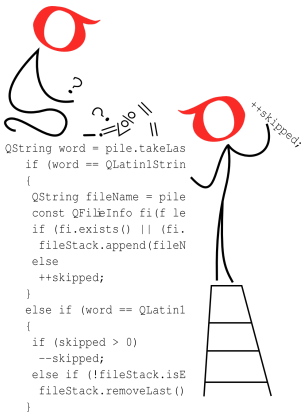


8 (3.34%)

Using these data, answer the following questions:

1. Which of those bases are likely to be essential components of the alien "DNA"?
2. How many different base pairs are in that "DNA"?
3. Assuming that the codon size is 3 (like in the Earth organisms), and they produce some protein-like molecules according to the same principle as we do, what is the maximal expected number of aminoacids in the alien "proteins"?

Computer Science



- Your program should be written in Java or Python-3
- No GUI should be used in your program: eg., easy gui Python
- All the input and output should be via files with specified in the problem statement
- Java programs should be submitted in a file with extension .java; Python-3 programs should be submitted in a file with extension .py.
  No .txt, .dat, .pdf, .doc, .docx, etc. Programs submitted in incorrect format will not receive any points!

  Please briefly describe your algorithm in the comments at the top of your program.

in

Problem 1. (5 pts)

Alice and Bob like to send messages to each other by slipping pieces of paper under each other's doors. Carl, who lives across the street, likes to impersonate Alice and Bob and slip fake messages under their doors. To combat the issue, Alice and Bob came up with a way to encode messages and verify that the message is genuine. When Alice or Bob wants to send a message, they first assign each symbol in the message a value.

- The alphabet A-Z is assigned 1-26
- The digits 0-9 are assigned 27-36
- "_" is assigned 37
- "!" is assigned 38
- "." is assigned 39
- "?" is assigned 40

Each symbol is also assigned a "weight," which corresponds to the symbol's position in the message. Once Alice or Bob have given each symbol a value and a weight, they multiply each value by its weight and add up the results to obtain a weighted sum. If the weighted sum is not divisible by 41, an extra character is appended such that the new weighted sum is divisible by 41. If the weighted sum is already divisible by 41, Alice or Bob sends the message as-is.

Let's say Alice is sending the message "HI_BOB!" to Bob. She converts the characters "H", "I", "_", "B", "O", "!" to the values 8, 9, 37, 2, 15, and 38 respectively, and calculates the weighted sum to be 8*1 + 9*2 + 37*3 + 2*4 + 15*5 + 2*6 + 38*7 = 498.
She notices that 498 leaves a remainder of 6 when divided by 41, and finds that when she appends "3" (which has a value of 30) to the end of the message, the new weighted sum is 738,

which is divisible by 41. Alice sends the message “HI_BOB!3”, and when Bob receives the message, he calculates that the weighted sum is divisible by 41, so Bob determines that Carl hasn’t tampered with the message. Alice and Bob are getting tired of encoding the messages by hand, so they’ve decided to infuse their messaging system with some modern technology.

Write a program that receives a message and if needed, append a symbol so the weighted sum is divisible by 41.
Your program should receive the input file **input.txt**, which will have one line containing the message, composed of uppercase letters (A-Z), the digits 0-9, and symbols ‘**_**’, ‘**!**’, ‘**.**’, ‘**?**’.

Example input file:

```
HELLO
```

Your program will produce the output file **output.txt**, which will contain the encoded message. If the weighted sum of the input is divisible by 41, output the message as-is, and otherwise output the message and append a character that makes the weighted sum divisible by 41.
Example output file:

```
HELLO5
```

Problem 2. (10 pts)

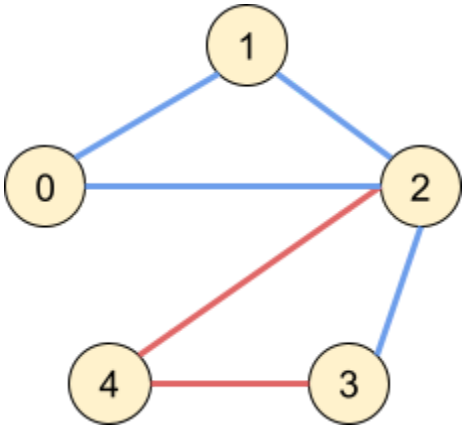
After his efforts to impersonate Alice and Bob failed, Carl dreamed bigger and has now taken over the railroad system of SigmaLand. SigmaRail has *s* stations with *t* stretches of track, each stretch connecting two stations. Each station has been assigned a number from 0 to *s*-1 that is used to issue tickets. Each ticket allows a SigmaRail customer to travel one stretch of tracks. As part of his evil plan, Carl has color-coded each stretch of tracks to be either blue or red, and in order to require customers to buy the maximum amount of tickets, he insists that all customers alternate their routes between blue and red tracks. Alice and Bob have grown tired of Carl’s rail system and bulk ordered red and blue paint to switch the colors of the rail tracks and make their trips on SigmaRail cheaper. However, they must paint the rails at night to avoid being caught, so they only have time to switch the color of one stretch of rails.

Write a program that receives *s* (the number of stations), *t* (the number of stretches of track), the stations each stretch of track connects to, and the color of each stretch of track, and determines which stretch of track Alice and Bob should switch the color of to minimize the number of tickets they need to buy if they starts at station 0 and want to arrive at station *s* - 1. Additionally, your program should calculate the shortest alternating path from station 0 to station *s* - 1 resulting from the change.

Your program should receive the input file **input.txt**, which will consist of $s + 1$ lines. The first line will contain s and t, and all subsequent lines will contain the stations each stretch of track connects to and its color.
Example input file:

```
5 6
2 3 b
4 3 r
2 1 b
1 0 b
2 4 r
2 0 b
```

This input would correspond to the following train map:



Your program will produce the output file **output.txt**, which will contain two lines. The first line will contain the two stations connecting to the stretch of track Alice and Bob should switch the color of, with lower number going first, and the second line will contain the least amount of tickets they need to buy to find an alternating path after the change. If Alice and Bob shouldn't switch the color of a stretch of tracks, write "NONE" on the first line. If there is no possible path, write "IMPOSSIBLE" on the first line and leave the second line blank.
Example output file:

```
NONE
2
```

Alice and Bob shouldn't change the color of any stretches of track. They can reach their destination by buying two tickets of alternating color.

Linguistics

Problem 1. (5 pts)

In Tongon, there are separate terms to refer to a common person or object and higher powers. For example, Houhau means “redfish with poisonous spikes” when speaking in a common context. However, when referring to gods, houhau means “be angry.” You can trace the logical progression–a person’s face turns red when they are angry.
The following is a mixed list of pairs of common and godly words and their definitions. Identify which common terms evolve to which god terms.

1. **Malu** = mild (referring to wind or sea)
2. **Toka** = (for a boat) to come aground
3. **Fakahaue** = stroll
4. **Langi** = burial place
5. **Ha’ele mai** = to come
6. **Tupu** = spring up
7. **Halifoea** = to suffer from hunger at sea
8. **Ha’ele** = to toddle
9. **Fakamalu** = have a bath
10. **Langi** = sky, heaven
11. **Toka** = sleep
12. **Halofia** = be hungry
13. **Tuputamaki** = be angry
14. **Haue** = cool and airy

Problem 2. (10 pts)

Tongon is spoken on Tonga, one of the Pacific Islands. Since they are both geographically close and isolated from one another, each island has developed distinct but related languages. Below are three sentences in Tongon and their English translations.

- **‘Okú nofo’i ‘a tangatá ia hake** = The man sits on the hill
- **Ko e fetu’ú ia koe langi** = The star is in the sky
- **Ko e ia lahi mähiná** = The moon is big

These three Tongan sentences are each translated into three distinct Polynesian languages (A, B, and C) below.

1. Kei tangata te puke noho-ia
2. Hei lani ko hōkū
3. Ne nofo e taane he tike
4. He nui ko malama
5. He whetuu i a rangi
6. Lahi teva e malama

7. I noho ia nei pu'u ko kāne
8. E fetū loto he lagi
9. He marama rahi

For each sentence, indicate which English sentence it translates to and identify which other two sentences are in the same language. For example: *El hombre se sienta en la colina* = The man sits on the hill = Language A

Which language is most closely related to Tongon, which one is second most related, and which one is least related?

The application deadline is April 15, 2023. We will notify applicants regarding acceptance decisions no later than **May 1st**.

Good luck with your application!