POM

SigmaCamp's *Problem of the Month* Contest

NOVEMBER 2025

Change for November 2025

Video submissions may now be at most **2.5 minutes long** for 5pt/10pt problems, and at most **6 minutes long** for projects.

The **POM Office Hour** is on **Saturday, December 6th** at **12PM EST**. Come ask us about the problems!
See sigmacamp.org/ pom/office-hours for details.

Video Submissions

Starting this year, **all solutions must be accompanied by short videos explaining your work.**

- Videos must be at most **2.5 minutes long** for 5pt/10pt problems, and at most **6 minutes long** for projects.
- **Solutions must be narrated**, but you do not need to show your face. Acceptable formats include:
  - Screensharing slides or a drawing app (e.g., MS Paint) with narration.
  - Recording a whiteboard, paper, or easel with narration (contents may be pre-written).
  - Speaking directly to the camera.
- **Submit videos as links** (Google Drive, Youtube, Dropbox, etc.). Extra requested files may be submitted as a single PDF file per problem.
- For coding problems, submit your code along with a video explaining your submission. Only **Python-3** (`.py`) and **Java** (`.java`) submissions are accepted.

Please see sigmacamp.org/pom for full details on the 2025 POM format change.

Project (30 points)

Linguistics

Introduction

In this project, you will be constructing your own spoken language, or as nerds on the Internet call it, *conlanging*. Points will be given for a combination of factors including your creativity, linguistic consistency, and explanation quality.

NOTE: Part (a) should not be explained in the video but only in a written component. For parts (b) and (c) all sentences in your language must be in the written component AND read aloud in the video.

Note that "spoken language" specifically excludes sign languages and other non-spoken varieties of language.

(a) **Language info sheet (WRITTEN ONLY):**
Create a language info sheet that explains the sounds, lexicon, syntax and history of your language.

i) Sounds: All languages start with sounds. Use the IPA (if you're unfamiliar, use the IPA Chart) to describe your consonants and vowels.

ii) Lexicon: Write some words in your language and their English translations. Think of what words are important in your language. For ideas of important words, see this list. Remember to include the words that you use in other parts of the project. Does your language have any interesting word structures? What about conjugations or inflections?

iii) Syntax: Explain the structure of sentences in your language. What are the rules that govern how words are combined together?

iv) History: Tell us about the history of your language. Explain how your language's origin, evolution, and developmental context have influenced its current sounds, syntax and lexicon.

(b) **Linguistics Problem Creation (WRITTEN + VIDEO):**
Create your own linguistics problem including 5–8 phrases in your language and a scrambled list of their English translations. The goal of the problem should be to match these phrases to their translations.

Make sure there is some way for your problem to be solved without prior knowledge of your language. You can include clues such as root words, plurals, parts of the word that indicate part of speech, etc.

Then, write and record a solution for your problem including the logical steps you'd expect someone to take in order to match each phrase to its translation.

(c) **Poetry (WRITTEN + VIDEO):**
Write a short (3–6 lines) piece of poetry written in a characteristic style of your language. Poetry varies across cultures and languages—think about limericks, haikus and nursery rhymes.

In your video, read your poetry and then translate it into English with an explanation of how its structure works and connects to its meaning.

Hint:

No hint this month.

Solution:

Featured Solution
by Eva Fridman
https://youtu.be/EwQEQKigKk0

Featured Solution
by Anya Labrecque
https://youtu.be/tpLlErT_61E

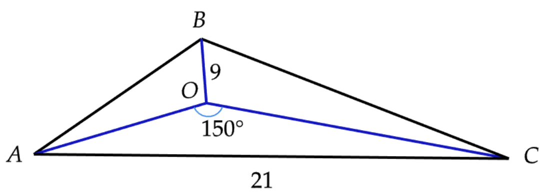
Mathematics

For all mathematics problems, please provide full justification. **Do not include any code** in your submission unless specified otherwise — all code submissions will be awarded no points.

Please show any diagrams or equations that you reference in your video, or attach them as a separate PDF.

5 points:

You are exploring a mysterious triangular island with corners labeled A, B, and C. In the *incenter* of the island is a flag, labeled O, and three straight trails lead from O to the corners A, B, and C. Each trail splits the corner it starts from into two equal angles (i.e., the trails are *angle bisectors*). Here is a picture of the island (not to scale):



An old pirate map provides you with the following hints:

- The distance between corners A and C is 21 miles,
- The distance from the flag O to point B is 9 miles, and
- The angle at O between the trails OA and OC is $\angle AOC = 150°$.

Using these clues, find the perimeter of the island.

Hint:

Consider the properties of incenter.

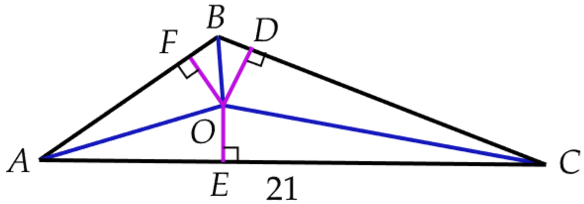
Solution:

Answer: 51

From $\angle AOC = 150°$ we can conclude that $\angle OAC + \angle OCA = 30°$, then from bisectors $\angle BAC + \angle BCA = 60°$, hence $\angle ABC = 120°$ and $\angle ABO = \angle CBO = 60°$.

Since the incenter is the center of the inscribed circle, we can draw radii $OD = OF = OE$, perpendicular to the corresponding sides. The angles in $\triangle OFB$ are $30°$, $60°$, and $90°$, and then $BD = \frac{1}{2}BO = 4\frac{1}{2}$.

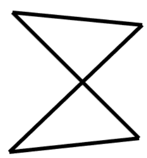
By HL theorem we have that $\triangle AOE \cong \triangle AOF$, $\triangle COE \cong \triangle COD$, and $\triangle OBF \cong \triangle OBD$. From this, $AF + CD = AE + CE = 21$ and $FB + BD = 9$, and the perimeter is $21 + 21 + 9 = 51$.



Featured Solution
by Ilana Afanasev
https://youtu.be/IrNQcIBFE-Q

10 points:

The picture below shows a self-intersecting quadrilateral, where one of its sides intersects another side. Note that the other two sides do not intersect.



Draw a self-intersecting hexagon in which **every** side intersects **exactly one** other side. Can this be done for a heptagon? Octagon? Nonagon? Decagon?

Hint:

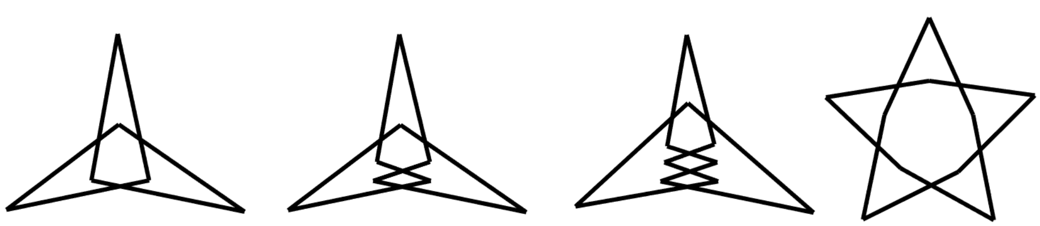
No hint this month. Just draw lots of pictures!

Solution:

The figure below shows the hexagon, the octagon, and the decagon. It should be clear from these 3 images how to construct a polygon with any even number of sides, with each side intersecting exactly one other.

The second decagon shows that for some numbers of sides it is possible to construct such a polygon from a star with half as many sides.

Since the condition "each side intersects exactly one other side" creates a pairing of the sides, it is impossible for a polygon with an odd number of sides to satisfy this condition.

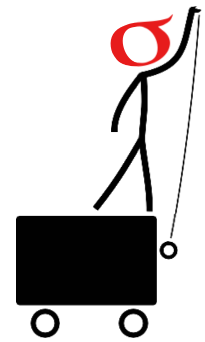


Featured Solution

by Alexander Miftakhutdinov

https://youtu.be/Dcd59IXuvoE

Physics



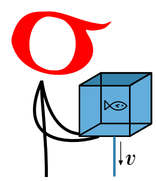
5 points:

A student is moving between apartments and is carrying a fish tank filled with water (the fish had the same density as the water and can be ignored). The fish tank develops a small leak, and water starts exiting the tank from the bottom at a speed v. Suppose the fish tank has the form of a cube with a side of $a = 20\,cm$.

(a) Determine the speed v of the water leaking out of the tank.

(b) Now imagine the student walks into an elevator with the fish tank. They notice that the leak slows down: the water now flows out *half as fast*. Determine the direction and the magnitude of the elevator's acceleration (assume the water level does not change much).

(c) How heavy would the fish tank feel to the student in the elevator compared to in the apartment? *Hint: You can find the actual mass of the tank, and compare it to its "effective mass" in the elevator.*



Some Background Information. In this problem, we will explore some fundamentals of modeling fluids. A particularly useful concept when describing the motion of a fluid is *pressure*, defined as

$$P = \frac{F}{A} \tag{1}$$

where P is the pressure on some region of area A, and F is the force acting on that region.

Let's see how pressure works in a static liquid. Consider a rectangular prism filled with an incompressible liquid of density ρ, and look at the force that a block of liquid of height h exerts on the liquid below it due to gravity.

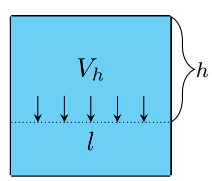


Figure 1: Side view of rectangular prism filled with a liquid of density ρ.

The volume of this block of liquid is $V = h \cdot A$, where A is the area of the bottom of the container in Figure 1. Since density is mass divided by volume (that is, $\rho = \frac{m}{V}$), we can determine that the force of gravity that pushes down on that block of water is

$$F_g = (\rho \cdot V) \cdot g = \rho \cdot h \cdot A \cdot g$$

where $g = 9.8\,\frac{\text{m}}{\text{s}^2}$ is the acceleration due to gravity on Earth's surface. We can use the definition of pressure in (1) to get that the pressure in a liquid is

$$P = \rho g h \tag{2}$$

where P is the pressure, ρ is the density of the liquid, h is the depth at which the pressure is measured.

Note that this pressure at a point of the liquid is exerted in all directions. The expression above then is the pressure felt by both the liquid and the walls of the container.

Hint:

No hint this month.

Solution:

(a) To determine the speed of the water leaking out of the tank in the apartment, we can use Bernoulli's equation. For the incompressible liquid, the following combination of quantities does not change along the flow:

$$\frac{\rho\, v_i^2}{2} + \rho\, g\, h_i = const, \tag{3}$$

where ρ is the density of the liquid, v_i is the flow speed at a certain point i, and h_i is the height of this point. Here, we assumed that the static pressure does not change along the flow. Of course, this equation is a consequence of energy conservation: the first term is the kinetic energy per unit volume of the liquid, and the second term is its potential energy. Assuming that the water does not move at the surface of the tank, we can compute the velocity of the leak at the bottom of the tank:

$$\rho\, g\, h = \frac{\rho\, v^2}{2}, \tag{4}$$

where $h = a = 0.2\, m$. This gives for v:

$$v = \sqrt{2\, g\, h} \simeq 2\, \frac{m}{s}. \tag{5}$$

(b) To understand why the water flow slows down in the elevator, let us first consider a free-falling body. The acceleration of this body is given by the second Newton law:

$$m\, A_{\text{free fall}} = m\, g, \tag{6}$$

where g is the gravitational acceleration. When the student walks into the elevator, he has to account for the acceleration of the elevator: the sum of the observed free-fall acceleration of the body and the acceleration of the elevator should sum up to the gravitational acceleration:

$$A'_{\text{free fall}} + A_{\text{elevator}} = g \quad \text{or} \quad A'_{\text{free fall}} = g - A_{\text{elevator}}, \tag{7}$$

where $A'_{\text{free fall}}$ is the observed free-fall acceleration in the elevator. If the elevator is accelerating up, then $A_{\text{elevator}} < 0$, and if it is accelerating down, then $A_{\text{elevator}} > 0$. Thus, we should replace g in all physical laws inside the elevator by $A'_{\text{free fall}}$. This is the famous equivalence principle postulated by Einstein, who assumed "the complete physical equivalence of a gravitational field and a corresponding acceleration of the reference system".

First, we will illustrate this principle by looking at the tank without the leak and computing the static pressure inside it. At the bottom of the tank (or at any fixed depth inside the tank), we can compute the normal force acting on the block of liquid using Newton's second law:

$$F_{\text{net}} = m\, g - N = m\, A_{\text{elevator}}, \tag{8}$$

which gives

$$N = m\, (g - A_{\text{elevator}}) = m\, A'_{\text{free fall}}. \tag{9}$$

By Newton's third law, the block of liquid is acting on the bottom of the tank (or on the block of liquid below) with a force of the same magnitude and opposite direction. Thus, in the elevator, the pressure inside the tank at a certain depth h is given by

$$P' = \frac{N}{S} = \rho\, h\, A'_{\text{free fall}}, \tag{10}$$

which can also be obtained by replacing g with $A'_{\text{free fall}}$ in the original formula for the pressure outside the elevator: $P = \rho\, g\, h$.

Now, we can apply the equivalence principle to Bernoulli's equation. Instead of (4), we have inside the elevator:

$$\rho\, h\, A'_{\text{free fall}} = \frac{\rho\,(v')^2}{2}. \tag{11}$$

This gives for the speed of the leak v':

$$(v')^2 = 2\,h\,A'_{\text{free fall}} = 2\,h\,(g - A_{\text{elevator}})\,. \tag{12}$$

Since the leak is slower in the elevator, we have $A_{\text{elevator}} > 0$ and the elevator is accelerating downward. To find the magnitude of the acceleration, we use $v^2 = 2\,g\,h$ and substitute $v' = v/2$ into (12):

$$\frac{1}{4} \times 2\,g\,h = 2\,h\,(g - A_{\text{elevator}})\,. \tag{13}$$

Solving for A_{elevator}, we get

$$A_{\text{elevator}} = \frac{3}{4}\,g. \tag{14}$$

(c) Similarly to part (b), we can compute the normal force acting on the fish tank from the hands of the student:

$$F_{\text{net}} = M\,g - N_{\text{tank}} = M\,A_{\text{elevator}}, \tag{15}$$

where M is the mass of the tank. Since $A_{\text{elevator}} = \frac{3}{4}\,g$, we have

$$N_{\text{tank}} = \frac{1}{4}\,M\,g. \tag{16}$$

Thus, the tank feels four times lighter in the elevator compared to in the apartment.

Answer: (a) $\sqrt{2\,g\,a}$; (b) $\frac{3}{4}\,g$, down; (c) 4 times lighter.

10 points:

For this problem, we will introduce rudimentary statistical mechanics. This is meant as an introductory problem, accessible even if you have not yet seen any statistical mechanics — the main requirement is an understanding of probability.

The central point of statistical mechanics is that we no longer think of a system in any one specific state, but as having a collection of probabilities of being in various states. For example, you may not know the precise location of a particular molecule of gas in the room you are currently sitting in, only the rough region it *might* be in. If we have many, many molecules of gas (around 10^{26} in your room), then we can average over the probabilities and find extremely precise results for the state of the entire body of gas.

A good approximation for the probability of a system to be in a given configuration was determined by Boltzmann – if the system is at a temperature T (in units of Kelvin), and a given configuration has energy E, then the probability p for the system to be in this state is proportional to

$$p \propto \exp\left(-\frac{E}{k_B \cdot T}\right). \tag{17}$$

where "$\propto$" means "proportional to", $\exp(x)$ denotes e^x, and k_B is Boltzmann's constant. The overall constant of proportionality is determined by requiring the sum of all probabilities be equal to 1.

As a simple example, suppose that a ball of mass m can be in two configurations: on a table of height h (where it has energy mgh), or on the floor (where it has energy 0). At temperature T, its probabilities are (remembering that $e^0 = 1$):

$$p_{\text{floor}} = \frac{1}{1 + \exp\left(-\frac{mgh}{k_B T}\right)}, \quad p_{\text{table}} = \frac{\exp\left(-\frac{mgh}{k_B T}\right)}{1 + \exp\left(-\frac{mgh}{k_B T}\right)}. \tag{18}$$

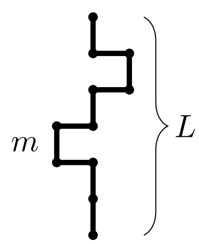
The *expected* height $\langle h \rangle$ (also the average height) is given by a weighted sum:

$$\langle h \rangle = 0 \cdot p_{\text{floor}} + h \cdot p_{\text{table}} = \frac{h \exp\left(-\frac{mgh}{k_B T}\right)}{1 + \exp\left(-\frac{mgh}{k_B T}\right)}. \tag{19}$$

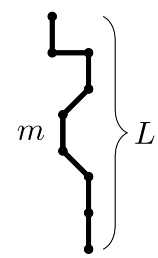
For some resources, see this short video or the first few pages of David Tong's notes (reading these is not necessary to solve the question).

Part (a) (1 point). What happens to the average height of the ball from the example above when the temperature goes to zero? How about when the temperature goes to infinity?

For the remainder of this problem, we consider a simple model for a piece of string hanging from a ceiling. We model it as a chain of N rigid rods with attached masses m. All the rods are of equal length h. They are connected by hinges that can swing freely (see Figure 2a). For simplicity, assume that each rigid rod can only be in three configurations: horizontal pointing left, horizontal pointing right, or hanging exactly vertically.



(a) For parts (b)-(c): The diagram above shows one possible configuration of $N = 10$ rods (restricted to be either horizontal or vertical), giving the string a total length L. Note that different configurations could have different lengths (i.e. imagine a configuration having all rods vertically down).



(b) For part (d): Model of a string as N rigid rods of mass m. In part d we allow the rods to be vertical, horizontal and at 45° angles. Note that all the rods are the same length! In the diagram above, we take $N = 8$. Once again, different configurations could have different lengths.

Note that we can describe the length configuration of the system, by specifying the vertical component of each rod. Let $h \cdot \Delta x_i$ be the horizontal component of the ith rod, where Δx_i can either be 1 or 0.
We will assume the convention of numbering rods from bottom to top. So for example in Figure (2a) $\Delta x_1 = 1$, $\Delta x_3 = \Delta x_5 = 0$ etc.

Part (b) (2 points). Suppose the system is in some given configuration with known $\{\Delta x_1, ..., \Delta x_n\}$. Write down the potential energy for this system. Show that the probability for the system to be in this configuration is equal to

$$P(\{\Delta x_1, ..., \Delta x_n\}) = \frac{1}{Z} \exp\left(\frac{mgh}{k_b T} \sum_{l=1}^{n} l \cdot \Delta x_l\right)$$

where Z is some proportionality coefficient.

Part (c) (3 points) By demanding that the sum of probabilities over all configurations must be 1, show that:

$$Z = \prod_{l=1}^{n} (2 + 1e^{\frac{mghl}{k_b T}}) = (2 + e^{\frac{mgh}{k_b T}}) \cdots (2 + e^{\frac{mghl}{k_b T}}) \cdots (2 + e^{\frac{mghn}{k_b T}})$$

Write down the expression for the probability that the lth rod is horizontal. Write down the probability that it is vertical. (*Note that when considering the probability that the lth rod is in some configuration, you need to also include the cases where all other rods are in any possible position.*)

Part (d) (3 points). What is the expected length of the dangling rope as a function of temperature? Write this down in terms of m, g, h and n. What is it at zero temperature? What happens to the rope as the temperature is increased?

Part (e) (1 points). Suppose the rods could now be vertical, horizontal and at 45° angles (see Figure (2b)). What are now the allowed values of Δx_i? Explain how solving the problem changes. Are the qualitative results obtained in part d) the same?

Hint:

No hint this month.

Solution:

Part a) When the temperature goes to 0, we see that $\frac{mgh}{k_bT} \to \infty$. This means that $\exp(-\frac{mgh}{k_bT}) \to 0$. From this we can conclude that the average height $\langle h \rangle = \frac{h\exp(-\frac{mgh}{k_bT})}{1+\exp(-\frac{mgh}{k_bT})} \to h\frac{0}{1+0} = 0$. I.e. at 0 temperature, the ball is on the ground.
As the temperature becomes arbitrarily large, we have $\frac{mgh}{k_bT} \to 0$ (T gets large, so $1/T$ is small). From this we conclude that $\exp(-\frac{mgh}{k_bT}) \to 1$ (as $\exp(0) = 1$). We then compute that the average height goes to $\langle h \rangle = \frac{h\exp(-\frac{mgh}{k_bT})}{1+\exp(-\frac{mgh}{k_bT})} \to h \cdot \frac{1}{1+1} = \frac{h}{2}$. At high temperature, on average the ball is half way between the ground and the top of the table.

Part b) First let's consider a system with only one rod (i.e. $N = 1$). In this case, we have 3 possible configurations. The rod is horizontal with mass on left, rod is horizontal with mass on right and rod is vertical with mass at the bottom. These three configurations have the following energies:

$$E_l = 0 \quad E_r = 0 \quad E_d = -mgh$$

We then get that the probabilities of the rod being horizontal left, right and vertical should be proportional with the same coefficient to:

$$P_l \propto 1 \quad P_r \propto 1 \quad P_d \propto \exp\left(\frac{mgh}{k_BT}\right)$$

Since we want the probabilities to add up to 1, we have:

$$\frac{1}{Z}\left(1 + 1 + \exp\left(\frac{mgh}{k_BT}\right)\right) = (P_l + P_r + P_d) = 1$$

We then get that $Z = 2 + \exp\left(\frac{mgh}{k_bT}\right)$. From this we can write down the probabilities to be:

$$P_l = P_r = \frac{1}{2 + \exp\left(\frac{mgh}{k_bT}\right)} \quad P_d = \frac{\exp(\frac{mgh}{k_bT})}{2 + \exp\left(\frac{mgh}{k_bT}\right)}$$

Now let's consider a system with N rods. Let Δx_l denote the vertical component of the lth rod with the first rod being the one at the bottom. We then get that the potential energy of the last rod is $E_N = -mgh\Delta x_1$. Note that the position of the next rod, depends on the position of the previous rod. This means that $E_{N-1} = -mgh(\Delta x_1 + \Delta x_2)$. We continue this pattern to see that the potential energy of the $N-k$th rod is given by

$$E_{N-k} = -mgh\sum_{l=N-k}^{N} \Delta x_l$$

The total energy of some given configuration is then:

$$E = -mgh \sum_{i=1}^{N} \sum_{k=i}^{N} \Delta x_k$$

It is easy to notice that in the sum above Δx_i appears exactly i times and we can rewrite the sum above as:

$$E = -mgh \sum_{i=1}^{n} i \cdot \Delta x_i$$

Now using the Boltzman formula, we get that the probability of obtaining a state in configuration $\{\Delta x_1, ..., \Delta x_N\}$ is:

$$P_{\{\Delta x_i\}} = \frac{1}{Z} \exp\left(\frac{mgh}{k_b T} \sum_{l=1}^{N} l \cdot \Delta x_l\right)$$

Part c)
We now want to compute Z in the expression above. As before, we will demand that the sum of probabilities over all possible configurations is 1.

$$\frac{1}{Z} \sum_{\{\Delta x_i\}} \exp\left(\frac{mgh}{k_b T} \sum_{l=1}^{N} l \cdot \Delta x_l\right) = 1$$

We compute this sum, by realizing that

$$\exp\left(\frac{mgh}{k_b T} \sum_{l=1}^{N} l \cdot \Delta x_l\right) = \prod_{l=1}^{N} \exp\left(\frac{mgh}{k_b T} l \Delta x_l\right)$$

We can now iteratively sum over all configurations, by first fixing the options of Δx_1, Δx_2 and so forth:

$$Z = \sum_{\{x_l\}} \exp\left(\frac{mgh}{k_b T} \sum_{l=1}^{N} l \cdot \Delta x_l\right) = \left(\sum_{\Delta x_1} \exp\left(\frac{mgh}{k_b T} \Delta x_1\right)\right) \sum_{\{\Delta x_2, \ldots \Delta x_N\}} \exp\left(\frac{mgh}{k_b T} \sum_{l=2}^{N} l \cdot \Delta x_l\right)$$

$$Z = (2 + e^{\frac{mgh}{k_b T}}) \cdots (2 + e^{\frac{mghl}{k_b T}}) \cdots (2 + e^{\frac{mghn}{k_b T}}) = \prod_{l=1}^{n} (2 + e^{\frac{mghl}{k_b T}})$$

Now let's write down the probability that the lth rod is vertical. We therefore need to sum over all of the probabilities of configurations where $\Delta x_l = 1$

$$P(\Delta x_l = 1) = \frac{1}{Z} \exp\left(\frac{mghl}{k_b T}\right) \sum_{\Delta x_1, \ldots, \Delta x_{l-1}, \Delta x_{l+1}, \ldots, \Delta x_N} \exp\left(\frac{mgh}{k_b T} \sum_{k \neq l} k \Delta x_k\right)$$

Using the same ideas as above, when we computed Z, we can write that sum as a product that is missing the term $(2 + e^{\frac{mgh}{k_b T} l \Delta x_l})$. Substituting the expression for Z, we get that these factors cancel and we are left with:

$$P(\Delta x_l = 1) = \frac{\exp\left(\frac{mgh}{k_b T} l\right)}{2 + \exp\left(\frac{mgh}{k_b T} l\right)}$$

In a similar fashion, we can compute that:

$$P(\Delta x_l = 0) = \frac{2}{2 + \exp\left(\frac{mgh}{k_b T} l\right)}$$

Part d) Before we compute the average length of the whole string, let's compute the average vertical length of the lth rod. We do this via:

$$\langle h\Delta x_l \rangle = 0 \cdot P(\Delta x_l = 0) + h \cdot P(\Delta x_l = 1) = h \cdot \frac{\exp\left(\frac{mgh}{k_b T} l\right)}{2 + \exp\left(\frac{mgh}{k_b T} l\right)}$$

We then see that the expected length of the whole string, should be the sum of expected lengths of its components:

$$\langle L \rangle = h \sum_{l=1}^{N} \frac{\exp\left(\frac{mgh}{k_b T} l\right)}{2 + \exp\left(\frac{mgh}{k_b T} l\right)}$$

Now let's explore the low and high temperature limits. If $T \to 0$ then $\frac{\exp\left(\frac{mgh}{k_b T} l\right)}{2+\exp\left(\frac{mgh}{k_b T} l\right)} \to 1$. We then get that:

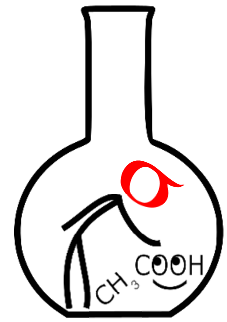
$$\langle L \rangle \xrightarrow{T \to 0} hN$$

Sow at low temperature the string is fully extended.
For the high temperature limit as T gets large , $\exp\left(\frac{mgh}{k_b T} l\right) \to 1$. Then $\frac{\exp\left(\frac{mgh}{k_b T} l\right)}{2+\exp\left(\frac{mgh}{k_b T} l\right)} \to \frac{1}{3}$
We then get that:

$$\langle L \rangle \xrightarrow{T \to \infty} \frac{hN}{3}$$

This means that at high temperature, the string is contracted to a third of its length.
Part e) The problem changes only slightly. Two new configurations are added where now Δx_i has 2 configurations where it is 0, 2 configurations where it is $\frac{1}{\sqrt{2}}$ and 1 configuration where it is 1. The calculation is otherwise the same. The low temperature limit is the same (string is fully extended). High temperature limit will change, as now more options are possible.

Chemistry



5 points:

There are four acids that all fit the formula H_nXO_m, with similar molar masses (within ± 2.5 g/mol from the average). In each acid, the central atom is bonded to the same number of surrounding atoms, but as the molar mass increases, the total number of bonds increases, and so does the acidity. Identify those acids, draw their structures, and explain why these patterns occur.

Hint:

When dealing with a series of chemical compounds whose molecular masses are similar and whose numbers of chemical bonds change only slightly, what can be said about the relevance of the Periodic Table to this issue?

Solution:

Answer: H4SiO4, H3PO4, H2SO4, HClO4.

The molecular formula of a ternary acid is H_xEO_y, where E is a non-metal or semimetal. If the molecular masses are similar, the number of oxygens must be the same and the molar mass of E has to be very similar, hence they should be contiguous atoms. Inspecting the periodic table, this is true for Si, P, S and Cl. It cannot be Ge, As, Se, and Br, because the mass of H_2SeO_4 is greater than the mass of $HBrO_4$.

The ternary options for each elements are:

- Si: H_4SiO_4, H_2SiO_3, $H_6Si_2O_7$, $H_2Si_2O_5$;
- P: H_3PO_4, H_3PO_3, H_3PO_2, HPO_3, $H_4P_2O_7$, $H_4P_2O_5$
- S: H_2SO_4, H_2SO_3, $H_2S_2O_7$, $H_2S_2O_3$, H_2SO_5, $H_2S_2O_8$
- Cl: $HClO_4$, $HClO_3$, $HClO_4$.

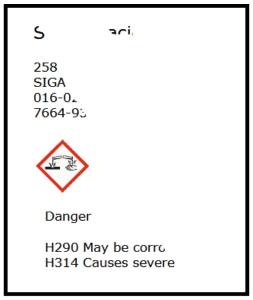
Comparing the molar masses and the number of oxygens, the only combination is: H_4SiO_4, H_3PO_4, H_2SO_4, $HClO_4$. The number of bonds increases because the oxidation number of the central atom increases. Because the coordination number is the same (4), the increase in the bond order is reflected as an increase of double bond character from H_4SiO_4 (0), to $HClO_4$ (3). The increase of oxidation number of the central atom makes ionization of H^+ easier, increasing the acidity.

Featured Solution
by Hanson Jiang
https://youtu.be/CfdZVK0JOQQ

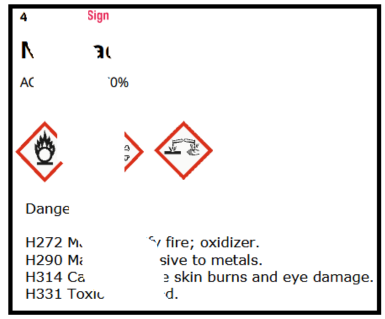
10 points:

While cleaning the lab, Alice discovered three glass bottles and one polyethylene bottle at the back of a chemical storage cabinet. Unfortunately, the labels on the bottles were damaged (see Figures 1-4).

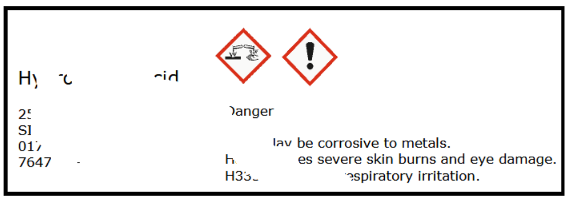
Alice wants to determine which chemical is in each bottle by reacting them with zinc metal, copper foil, and platinum wire. She determined:



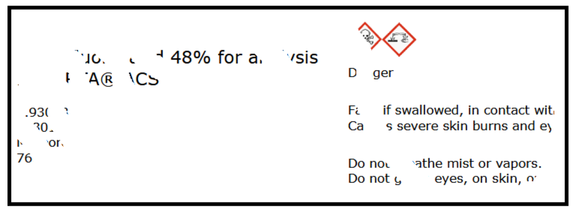
(a) Chemical 1



(b) Chemical 2



(c) Chemical 3



(d) Chemical 4

- Chemical 1 reacted strongly with zinc, but did not dissolve copper until it was heated, at which point the copper began to react slowly, forming a light blue solution.
- Chemical 2 reacted vigorously with both zinc and copper, releasing dark brown fumes.
- Chemical 3 reacted vigorously with zinc, but did not react visibly with copper, even when heated.
- Chemical 4 (in the polyethylene bottle) reacted slowly with zinc and did not react visibly with copper. Alice left the glass beaker containing copper and Chemical 4 on the bench overnight to see whether a slow reaction would occur. When she returned, she found that the chemical had etched through the beaker, and all of the liquid had spilled onto the table.

As Alice expected, platinum did not react with any of the chemicals. However, when she mixed chemicals 2 and 3, platinum reacted with this mixture quite actively.

What compounds is each bottle likely to contain? What other tests could be used to confirm their identities?

Hint:

Alice took those bottles from the shelf where she usually keeps acids.

Solution:

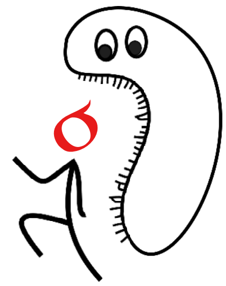
Answer: Usually, but not always, if some chemical reacts with metals, that chemical is most likely an acid. Normal (non-oxydizing) acids react with metals left to hydrogen in the metal reactivity series. That is consistent with what preserved parts of labels say: all those compounds are highly reactive and highly corrosive, and it would be correct to conclude they are acids. But which is which? The bottle number 3 contains an ordinary strong acid: it does not react with copper, although it reacts vigorously with zinc. Importantly, it is not toxic (there is no "Toxic" pictogram on the label). Most likely, it is hydrochloric acid, HCl, but we cannot tell for sure so far. The bottle #1 most likely contains sulfuric acid, H_2SO_4, which is a strong acid too, but it is also a mild oxidizer: a hot H_2SO_4 reacts with copper, because sulfur oxidizes copper, and sulfur dioxide is released. The hypothesis that that bottle contains H_2SO_4 is consistent with the fact that the chemical is not toxic (according to the laber): H_2SO_4 is corrosive, but it by no means toxic. The last bottle (number 4) is also easy to identify: usually, only hydrofluoric acid (HF) is stored in plastic bottles, because it reacts with glass. In addition, it is a relatively weak acid, and it reacts with zinc only (it cannot react with copper), and does that slowly. Finally, the acid that reacts both with zinc and copper at room temperature and produces a brown fumes (the bottle number 2) is most likely nitric acid, HNO_3. That happens because nitric acid is a strong oxidizer, and in a reaction with copper, the nitrogen atom in HNO_3 takes electrons from copper, so the copper metal becomes a copper ion, Cu^{2+}, and, simultaneously, the pentavalent nitrogen in HNO_3 becomes NO. The latter is a colorless gas, but when it comes to a contact with air, it is quickly oxidized to yield a brown NO_2. But why does brown gas forms in a reaction with zinc? The reason is that HNO_3 is a strong oxidizer, and it simply does not allow hydrogen to form: it oxidizes H_2 molecules immediately after they form, and the only product is the same brown gas as in the reaction with copper (i.e. NO_2).

But the final argument that proves the guess about the bottles 2 and 3 is the experiment with platinum. This metal is called "a noble metal", because it doesn't rust and doesn't react with virtually anything. But there is an exception: *aqua regia*. This Latin term denotes the mixture of nitric and hydrochloric acid. This mixture was discovered by alchemists, and they gave this name (literally, "a royal water"), because it is capable of reacting even with the "king of all metals" (i.e. platinum and gold).

HCl and HNO_3 react with each other and produce nitrosyl chloride, ClNO, an extremely reactive compound that oxidized even platinum.

So, the conclusion is:

1. H_2SO_4
2. HNO_3
3. HCl
4. HF



Biology

5 points:

Why did many organisms develop to be aerobes when, during the emergence of life, they did not require oxygen at all? Why did some not evolve this ability?

Hint:

Compare the conditions on Earth during the Archean era with those of the modern world.

Solution:

The single greatest event that changed the course of atmospheric and ecological history on Earth was the Great Oxidation Event (GOE), where the atmosphere and bodies of water experienced an increase in levels of free oxygen. This was due to metabolic activity from photosynthetic organisms. Prior to this, early life survived under anoxic conditions and relied on its anaerobic metabolism for energy production, for which oxygen was not necessary.
Oxygen is a relatively reactive molecule that can oxidize/modify other molecules, including lipids, proteins, and enzymes' redox centers (iron-sulfur clusters, etc.). As oxygen became more abundant, anaerobic organisms faced the toxic properties of oxygen and developed systems to neutralize them. So, these organisms can occupy low-oxygen niches and persist through transient oxygen exposure. Since oxygen was present anyway, at a certain point in evolution, it allowed for aerobic respiration (oxygen served as the terminal electron acceptor in the Electron Transport Chain). Organisms that evolved this ability to use oxygen gained a major energetic advantage. This is because aerobic respiration produces significantly more ATP than anaerobic pathways.
The reason as to why some organisms did not evolve this ability is that there was no evolutionary pressure for organisms that reside in low-oxygen environments. These extremophiles use alternative electron acceptors, like sulfate or nitrate, in their electron transport chains. These electron acceptors yield less energy than aerobic respiration, but are sufficient for survival in these very niche environments. There are incredible few multicellular animals that have been discovered that go their entire lives without oxygen

For more information about the Electron Transport Chain, visit this textbook or this Khan Academy lesson. This article by Nick Lane also provides a great overview of the role of oxygen in the evolution of complex life.

10 points:

During the early stages of evolution, when there was no oxygen, there were still plenty of energy sources. Later, as all old sources of energy became depleted, the biosphere switched to solar light as the major source of energy. The predominant mechanism of acquiring energy became photosynthesis, accompanied by the production of large amounts of oxygen as a byproduct. In this situation, three groups of organisms resorted to three different strategies of adaptation to these new conditions. Name these groups and explain what these strategies consisted of.

Hint:

Usually, when you encounter a new situation that requires changing your way of life, three common responses emerge: (i) pretend nothing has happened and continue living as before, (ii) adapt to the new challenges, or

(iii) seek help from those who have already adapted. Interestingly, these are the same strategies adopted by the three major groups of living organisms.

Solution:

The three groups of organisms are the domains of life: **Archaea**, **Bacteria**, and **Eukarya**. The various adaptation strategies that they have adopted are aerobic respiration (Eukarya), oxygenic photosynthesis (Bacteria), or staying the same (Archaea).

1. Eukarya exploited the newly abundant oxygen the most, evolving aerobic respiration (which is expanded on more in the 5pt solution). This evolutionary adaptation allowed for more efficient energy production, which enabled greater cellular complexity, supporting the evolution of complex organisms.
2. Bacteria were largely involved in the transition to a solar-powered biosphere, most notably through the evolution of oxygenic photosynthesis in cyanobacteria. Cyanobacteria used light energy to extract electrons from H_2O, allowing for carbon fixation using an effectively inexhaustible electron donor. A byproduct of this pathway was the release of free oxygen, which allowed for the oxygenation of the atmosphere and the oceans, eventually completely reshaping Earth's biochemistry and allowing for the evolution of multicellular, complex life.
3. Archaea largely persist in anaerobic niches and retain the basal pathways that rely on alternative electron acceptors in their electron transport chains. Through their occupancy of low-oxygen environments, they avoid oxygen toxicity and continue to thrive without the need to adopt oxygen-dependent metabolisms.

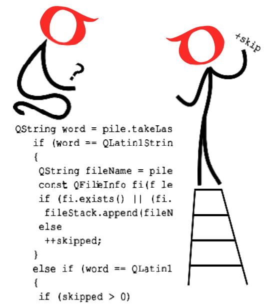
Simplified, these are the methods of adaptation:

i. Archaea, already adept at survival under extreme conditions, largely sidestepped this environmental shift by persisting in oxygen-poor or anoxic niches.
ii. Bacteria began producing oxygen via oxygenic photosynthesis and subsequently capitalized on atmospheric oxygen by evolving aerobic respiration as an efficient energy-yielding pathway.
iii. Eukaryota emerged as a composite adaptation in which archaeal hosts incorporated oxygen-producing and oxygen-consuming symbionts, enabling participation in an oxygenated biosphere through endosymbiosis rather than de novo metabolic invention.

It is important to note that there is a huge metabolic diversity in the Bacteria and Archaea today. There are many examples of anaerobic metabolisms in the Bacteria (e.g., sulfate-reducing bacteria, *Desulfovibrio sp.* and aerobic metabolisms in the Archaea (e.g., ammonia-oxidizing archaea, *Nitrosopumilus maritimus*). We consider the domains of life for the solution as a generalization and as a simpler way to think about these evolutionary strategies.

For more resources, click these links:

i. Oxygenic Photosynthesis
ii. Endosymbiotic Theory



Computer Science

- Your program should be written in Java or Python-3.
- No GUI should be used in your program (e.g. `easygui` in Python).
- All the input and output should be done through files named as specified in the problem statement.
- Java programs should be submitted in a file with extension `.java`; Python-3 programs should be submitted in a file with extension `.py`. **No `.txt`, `.dat`, `.pdf`, `.doc`, `.docx`, etc. Programs submitted in the incorrect format will not receive any points!**

When writing arithmetic expressions, we often write the symbol representing the operation in between its two arguments. For example, the sum of 1 and 2 is written as `(1 + 2)`. If we had three baskets with a pear and two apples in each basket, the total amount of fruit would be written as `(3 * (1 + 2))`. This notation, keeping the operator in between its arguments, is called *infix notation*.

It is also possible to place the operator symbol before the expression. Our two examples would be written as `(+ 1 2)` and `(* 3 (+ 1 2))`. This is called *prefix notation*. As you may have guessed, there is also *postfix notation*, which places the operator after the arguments. Our two examples would be `(1 2 +)` and `(3 (1 2 +) *)`.

5 points:

The remarkable thing about postfix and prefix notation, is that the parentheses are never necessary! There is only one way to parse a postfix or infix expression even without parentheses. Due to this nice property, postfix expressions used to be popular for calculators.

For this problem, use the following formal definitions of postfix and infix. A postfix expression is either an integer, or an expression of the form "`A B op`" where `A` and `B` are postfix expressions and `op` is one of `+`, `*`, `-`, `/`. Thus, `34 9 +` is a valid postfix expression that evaluates to 43, and `34 9 + 2 -` is a valid postfix expression that evaluates to 41. An infix expression is defined similarly, but we need parentheses for the value to be well-defined. An in-fix expression is either an integer or "`(A) op (B)`", where `A` and `B` are in-fix expressions and `op` is one of the operators mentioned above. So `(34) + (9)` evaluates to 43, and `((34) + (9)) - (2)` evaluates to 41. We need the parentheses since, for example, `10 - 3 - 2` could evaluate to either 5 or 9, depending on how it is parenthesized. You can assume that all the numbers in the input are nonnegative integers.

Write a program that translates from infix to postfix. Your program should read the input file `input.txt`, which contains a single line with an infix expression. The program should write the corresponding postfix expression on the first line of the file `output.txt` (without actually evaluating any of the operators).

Sample `input.txt`:

```
((34) + (9)) - (2)
```

Sample `output.txt`:

```
34 9 + 2 -
```

Hint:

No hint this month.

Solution:

Featured Solution

by Leon Kempe Regev

https://www.youtube.com/watch?v=04F1_hhBPaM

10 points:

An engineer decides to write a prefix-notation calculator—a program to parse and evaluate arithmetic expressions in prefix notation—to use for their daily engineering calculations. As they use the calculator, they quickly grow tired of copying. The calculator has no memory, so intermediate values must be written down and typed back in if used in multiple places, and common approximations such as $\sin x \approx x - \frac{x^3}{6}$ have to be punched in again and again as (- x (/ (* x (* x x)) 6)), where each x has to be manually substituted! The engineer decides they would like to the ability to set variables and define single parameter functions within the calculator.

You are given input.txt which contains expressions on each line. Write a program which evaluates expressions line by line, keeping track of defined variables and functions, and writes the value of each input line on the corresponding line of output.txt. All expressions, even function and variable assignments, evaluate to an integer. Expressions are either:

1. Non-negative integers such as 12345 or 42.
2. Variables, which are strings of alphabetic characters, except the words "if", "set", and "defun" (these will be reserved operators). These evaluate to their most recently stored value. Examples of variable names include var and dolphin.
3. An application of an operator, which is of the form

```
(<op> <subexpr1> <subexpr2> ... <subexprN>)
```

   where the operator <op> is one of +, -, *, /, <, =, if, set, defun, or any allowable variable name, and each sub-expression <subexpr_> is an expression itself! Examples of operator application include

```
(+ 1 2)
(= (+ 1 2) 3)
(set dolphin (* (+ 1 2) (/ (* 6 7) 5)))
(if (= dolphin 0) (set dolphin 1) (set dolphin 0))
```

The number of sub-expressions depends on the operator. The operators are:

- The four integer operations +, -, *, and / allow 2 sub-expressions as arguments and apply the corresponding arithmetic operation. For division, return the floor of the quotient if the result is not an integer, so (/ 3 2) evaluates to $\lfloor \frac{3}{2} \rfloor = 1$.
- The two comparison operations <, = allow 2 sub-expressions as arguments and evaluate to 1 when true and 0 when false. For instance, (< 1 2) evaluates to 1, because 1 is less than 2.
- The set operation is always of the form (set <var> <expr>). The first sub-expression, <var>, is always a variable name, and the second, <expr> is any expression. The operation evaluates to the second sub-expression <expr> and stores its value. Instances of the variable on subsequent lines evaluate

to the stored value. For example, the expression (set dolphin (* 6 7)) stores and returns 42. If the next line is (+ dolphin 1), the sub-expression dolphin evaluates to 42, and the line evaluates to 43.

- The defun operation is always of the form (defun <func> <param> <body>) and always returns 0. Out of the three sub-expressions, the first two, <func> and <param> are always allowable alphabetic variable names. The expression <body> is any expression, which may contain instances of <param>. On subsequent lines, the expression (<func> <expr>) is an operation which takes a single sub-expression <expr> as an argument, and returns the value of <body> where each instance of <param> evaluates to <expr>. For example, the line (defun plusfive n (+ n 5)) defines the operator plusfive. A subsequent line (plusfive (* 2 2)) evaluates to 9, because the parameter n is set to $2 \cdot 2 = 4$, and (+ n 5) to evaluates $4 + 5 = 9$.
- The if operation is of the form (if <cond> <ontrue> <onfalse>), where <cond>, <ontrue>, and < onfalse> are arbitary expressions. The operator evaluates <cond>, and returns the value of <onfalse> if <cond> evaluates to 0, and the value of <ontrue> otherwise. Exactly one of <ontrue>, <onfalse> is evaluated, and variable and function definitions are only executed on the evaluated branch. For example the line (if 1 (set dophin 42) (set dolphin 43)) executes (set dolphin 42), which evaluates to 42. A subsequent line dolphin evaluates to 42, because the <onfalse> branch, (set dolphin 43), was never executed.

You may assume there are no variable, function, or parameter name conflicts in input.txt, i.e. you may ignore the issue of variable scoping; all variables and parameters may be global. The test cases will avoid any variable conflicts, and will always use novel names for parameters.

Sample input.txt:

```
(+ 1 2)
(defun plustwo n (+ n 2))
(= (plustwo 1) 3)
(set dolphin (* (plustwo 1) (/ (* 6 7) 5)))
(if (= dolphin 0) (set dolphin 1) (+ 1 (set dolphin 42)))
dolphin
(defun iseven a (= a (* 2 (/ a 2))))
(defun collatzstep k (if (iseven k) (/ k 2) (+ (* 3 k) 1)))
(collatzstep dolphin)
(collatzstep (collatzstep dolphin))
```

Sample output.txt:

```
3
0
1
24
43
42
0
0
21
64
```

Hint:

No hint this month.

Solution:

Featured Solution

by Daniel Makarov

https://www.youtube.com/watch?v=wIVx7frZByc