Physics 5 Pt

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The radius of the moon's orbit is about 400,000 km, and the circumference is about $800,000\pi$ km. Since the moon orbits the Earth in 28 days, the moon orbits with a speed of about

$$\frac{800,000\pi}{28} = \frac{200,000\pi}{7} \ \frac{\text{km}}{\text{day}}$$

If gravity were suddenly turned off, the moon would maintain its speed and move along a path that is tangent to its orbit. After one week, the moon would have traveled about $200,000\pi$ km along this tangent path. By the Pythagorean Theorem, the distance from the Earth to the moon after one week without gravity would be about

$$\begin{aligned}
d &= \sqrt{(400,000)^2 + (200,000\pi)^2} \\
d &= \sqrt{(2 \cdot 200,000)^2 + (200,000\pi)^2} \\
d &= \sqrt{(200,000)^2 (2^2 + \pi^2)} \\
d &= 200,000\sqrt{4+\pi^2} \\
d &\approx 744,838
\end{aligned}$$

Thus, the moon would be almost 745,000 km away from the Earth after one week.

Physics 10 Pt

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Suppose a ball of radius R, mass M, and moment of inertia $I = \frac{2}{5}MR^2$ is spun with angular velocity ϖ and placed on a horizontal surface with friction. At first the ball slides, but eventually it starts to roll without sliding.

Let t be the number of seconds it takes for the ball to start rolling without sliding. Let v_t be the linear velocity of the ball at time t, and let ϖ_t be the angular velocity at time t. Since the ball is rolling without slipping at time t, it must be the case that $\varpi_t = \frac{v_t}{R}$.

Once the ball is placed on the surface, the only (unbalanced) force F acting on the ball is due to (kinetic) friction. So, $F = \mu Mg$, where μ is the coefficient of (kinetic) friction between the ball and the surface, and g is the acceleration due to gravity.

Since $F = Ma$, the ball experiences linear acceleration of

$$a = \frac{\mu Mg}{M} = \mu g$$

while it is slipping. Since the linear velocity of the ball is initially 0, at time t we have

$$v_t = at = \mu gt$$

In addition to the linear acceleration, the ball experiences angular deceleration while it is slipping. The torque on the ball due to friction is $\tau = RF = R\mu Mg$. Since $\tau = I\alpha$, the ball experiences angular deceleration of

$$\alpha = \frac{R\mu Mg}{I} = \frac{R\mu Mg}{\frac{2}{5}MR^2} = \frac{5}{2}\frac{\mu g}{R}$$

Thus, at time t we have

$$\varpi_t = \varpi - \alpha t = \varpi - \frac{5}{2}\frac{\mu gt}{R}$$

Since $\varpi_t = \frac{v_t}{R}$, we can solve for t :

$$\begin{aligned}
\varpi_t &= \frac{v_t}{R} \\
\varpi - \frac{5}{2}\frac{\mu gt}{R} &= \frac{\mu gt}{R} \\
\varpi &= \frac{7}{2}\frac{\mu g}{R}t \\
t &= \frac{2}{7}\frac{\varpi R}{\mu g}
\end{aligned}$$

Finally, we can compute the linear and angular velocities of the ball when it stops sliding:

$$\begin{aligned}
v_t &= \mu g\left(\frac{2}{7}\frac{\varpi R}{\mu g}\right) = \frac{2}{7}\varpi R \\
\varpi_t &= \varpi - \frac{5}{2}\frac{\mu g}{R}\left(\frac{2}{7}\frac{\varpi R}{\mu g}\right) = \varpi - \frac{5}{7}\varpi = \frac{2}{7}\varpi
\end{aligned}$$

So, when the ball starts rolling without slipping, its speed is $\frac{2}{7}\varpi R$.